SHORT SELLING, DEATH SPIRAL CONVERTIBLES, AND THE PROFITABILITY OF STOCK MANIPULATION

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Abstract

The SEC recently adopted Regulation SHO to tighten restrictions on short selling and curb abusive short sales, including naked shorting masquerading as routine fails to deliver. This paper models market equilibrium when short selling is permitted and contrasts the equilibrium with and without manipulators among the short sellers. I explain how naked short selling can routinely occur within the securities clearing system in the United States and characterize its potentially severe market impact. I show how a recent securities innovation called floating-price convertible securities can resolve the unraveling problem and enable manipulative short selling to intensify.

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1. Introduction

Manipulative short selling has a long and colorful history that dates back to the origins of organized stock markets (Allen and Gale, 1992). Bernheim and Schneider (1935) describe how bear pools operated on the Amsterdam Stock Exchange during the late seventeenth century. Stock manipulators carefully timed their aggressive 'bear raids' to exert maximum selling pressure. The price declines attracted free riders, and the combined pressure on the prices of the targeted stocks produced virtually assured profits. The manipulators found that they could defeat any opposition by employing "tricks that only sly and astute speculators invent and introduce," such as planting false rumors about the target firm's precarious condition in the press (Bernstein and Schneider, 1935). When similar manipulation occurred on the London Stock Exchange in the early eighteenth century, the British parliament passed a law prohibiting short selling in 1734. The law was not repealed until 1860, and short selling was not specifically authorized under English law until 1893 (Bernstein and Schneider, 1935). Numerous histories document how these and other manipulative short selling techniques have been woven into the fabric of the stock market.[1]

[1] Bernstein and Schneider (1935), Sobel (1965), and Wycoff (1968) chronicle the history of stock market manipulation over several decades culminating in the 1920s and 1930s when manipulative short sellers organized into large investment pools to concentrate their short selling for maximum impact. Their descriptions of the manipulative techniques and the destabilizing impact of bear pools on the New York Stock Exchange in the 1920s and early 1930s are reminiscent of the Amsterdam Stock Exchange manipulations of the seventeenth century and the London Stock Exchange manipulations of the eighteenth century. Manipulative short selling was blamed for causing the Great Crash, although a subsequent Senate investigation found that other factors played a bigger role in causing the crash. These histories also describe how manipulative short selling techniques have evolved. House Report (1991) found that short sellers, sometimes including "short-selling partnerships [with] very substantial financial resources," were instigating SEC investigations to depress the prices of their targeted stocks. SEC (2003b) cites short selling abuses in proposing restrictions to curb naked short selling. Later in the paper I explain how floating-price convertibles are one of the most recent enablers of short sale manipulation.

The SEC defines a short sale as the "sale of a security that the seller does not own or that the seller owns but does not deliver. In order to deliver the security to the purchaser, the short seller will borrow the security, typically from a broker-dealer or an institutional investor."[2] The potential for abuse in short selling is a concern to market participants, regulators, and academics alike.[3] The SEC adopted Regulation SHO on July 28, 2004 to tighten the restrictions on short selling and curb abusive short sales, such as naked short selling (SEC, 2003b, 2004).[4] The SEC proposed new Regulation SHO in October 2003 because of growing concern that naked short selling masquerading as routine fails to deliver had impaired market efficiency:

> Many issuers and investors have complained about alleged "naked short selling," especially in thinly-capitalized securities trading over-the-counter. Naked short selling is selling short without borrowing the necessary securities to make delivery, thus potentially resulting in a "fail to deliver" securities to the buyer. Naked short selling can have a number of negative effects on the market, particularly when the fails to deliver persist for an extended period of time and result in a significantly large unfulfilled delivery obligation at the clearing agency where trades are settled. At times, the amount of fails to deliver may be greater than the total public float. In effect the naked short seller unilaterally converts a securities contract (which should settle in three days after the trade date) into an undated futures-type contract, which the buyer might not have agreed to or that would have been priced differently. The seller's failure to deliver securities may also adversely affect certain rights of the buyer, such as the right to vote. More significantly, naked short sellers enjoy greater leverage than if they were required to borrow securities and deliver within a reasonable time period, and they may use this additional leverage to engage in trading activities that deliberately depress the price of a security. (SEC, 2003b, pages 6-7.)

Used appropriately, short selling promotes market efficiency by eliminating overpricing (Diamond and Verrecchia, 1987, D'Avolio, 2002, Duffie, Garleanu, and Pedersen, 2002, and

[2] The short seller later repurchases the security in the market, presumably after its price has fallen, and returns it to the lender to close out the short position.

[3] House Report (1991) expresses Congress's concern that abusive short selling is impairing market efficiency and criticizes the SEC for its lax enforcement of the rules designed to prevent manipulative short selling.

[4] A 'naked' short sale occurs when the seller has neither borrowed the shares nor made an affirmative determination that they can be borrowed, which the securities laws require, before selling them. This failure to borrow the shares results in a 'fail to deliver' until the shares can be borrowed and delivered to the purchaser. Naked shorting also has a long history. Stedman (1905) provides colorful accounts of Jacob Little and other short sellers who amassed great fortunes in the nineteenth century through manipulative short selling. Little, nicknamed the 'Great Bear of Wall Street,' would naked short shares, spread rumors about the issuer's pending insolvency, and then cover his short position at the resulting depressed prices.

Jones and Lamont, 2002).[5] However, when left unchecked, short selling can artificially depress share prices and impair market efficiency (SEC, 2003b).[6] Whether short selling has this unintended effect depends on first, whether there are rules and regulations that prohibit potentially abusive behavior and second, whether regulatory enforcement is adequate to ensure that market participants obey these rules (SEC, 2003b).

Manipulation is the "intentional interference with the free forces of supply and demand."[7] A manipulative trading strategy corrupts the market's price formation process to generate a riskless profit (Jarrow, 1992). Market manipulation can be profitable when there is a difference between the price elasticities of purchases and sales that the manipulator can exploit. Stock market manipulators use a variety of devices, such as releasing false information about a company into the market,[8] and employing trading strategies that impede the price formation process, such as naked shorting, wash sales, matched trades, and painting the tape, all of which inject misleading trading information into the market, to move market prices in the direction that benefits the manipulator. Illegal short selling, such as naked shorting, can distort market prices by creating artificial supply-demand imbalances (Thel, 1994). Consequently, the securities laws in the United States proscribe various restrictions on short selling that are designed to constrain it so that it can not be misused to manipulate stock prices below the true asset value (Thel, 1994, SEC, 2003b, 2004).

[5] Lamont and Thaler (2003) and Ofek and Richardson (2003) furnish empirical evidence that the restricted supply of shares available for borrowing inhibited short selling and contributed significantly to the recent dotcom bubble.
[6] "New Rules to Put Squeeze on Shorts," *Wall Street Journal* (January 27, 2005): C5, quotes an assistant director in the SEC's Division of Market Regulation, who expresses concern that massive naked shorting could create an 'endless' supply of shares that "could drive down the price in an abusive or manipulative way." The article goes on to note that Regulation SHO stemmed from instances where the short position in a stock approached or even exceeded the firm's entire supply of outstanding shares.
[7] Pagel, Inc. v. SEC, 803 F 2d, 942, 946 (8th Circuit, 1986).
[8] Placing false notices on electronic bulletin boards in Internet chat rooms is an example of the type of manipulative behavior that is difficult for regulators to monitor.

Manipulation can occur when informed traders can take advantage of uninformed traders who must trade to meet their liquidity needs (Glosten and Milgrom, 1985, Kyle, 1985, 1989, Easley and O'Hara, 1987, Allen and Gale, 1992, Allen and Gorton, 1992). Allen and Gale (1992) examine trade-based manipulation, in which a trader can manipulate a stock's price upward simply by buying shares and then sell them at a profit even when the purchases do not cause any price momentum. Manipulation in their model does not require traders who take overt action to alter the value of the firm, inject false information into the market to move the price higher, or create a corner. Asymmetric information and the difference in the price elasticities of purchases and sales are the key factors. Uninformed traders are uncertain whether the buyer knows that the stock is undervalued or instead intends to manipulate the price upward. Purchases have a greater price elasticity than sales due to the greater information content of purchases when the sellers include liquidity traders. Uninformed liquidity traders have less freedom to time their sales, and so informed traders, such as corporate insiders, are able to profit by exploiting both their information advantage and the liquidity traders' timing disadvantage. When liquidity sales are more likely than liquidity purchases, a purchase conveys more information because it is more likely that the trader is informed. The share price elasticity with respect to purchases exceeds the price elasticity with respect to sales, and a pooling equilibrium can occur in which price manipulation is profitable.

My model is in the spirit of Allen and Gale (1992) but focuses on short sales. I include active traders (arbitrageurs), who turn out to be the critical enabling factor that facilitates manipulative short sales in market equilibrium. I assume that active traders are uncertain whether the seller knows that the stock is overvalued or instead intends to manipulate the stock price downward. They are less knowledgeable than informed investors or manipulators but more knowledgeable than uninformed traders. Active traders seek out information regarding the firm's

prospects and look for signals in the trading behavior of informed investors, such as corporate insiders. They sell in response to short sales by informed investors and manipulators, whom they mistake for informed investors, which allows manipulative short selling to be profitable.

Active trader selling can resolve the unraveling problem and allow profitable opportunities for manipulative short selling. The unraveling problem would rule out trade-based short sale manipulation if the market consisted only of informed traders and liquidity traders. It is more difficult to justify forced purchases than forced sales by liquidity traders, who presumably do not have the same pressing need to buy as to sell (Allen and Gorton, 1992). The asymmetry in price elasticities that creates an opportunity for manipulative purchases to be profitable rules out profiting from manipulative short sales. A manipulator can repeatedly buy stocks and then sell them to earn a profit because purchases having the greater price impact. But selling and then buying would have the opposite effects and result in a loss.

Active traders interact with the informed investor to create downward price momentum. Jarrow (1992) investigates how manipulation can occur when large trades create price momentum that leads to a difference between the price elasticities of purchases and sales. Price momentum occurs when trades are large enough to move the price and an increase in price at one date causes an increase in price at a later date. A large trader's purchases create upward price momentum, and then she trades against the price trend to lock in her profit by selling to noise traders who buy at the inflated price. Presumably this sort of manipulation could work in reverse with the large trader selling short to stimulate downward price momentum and then covering his short position by buying at depressed prices from noise traders. In my model active traders sell in the next period when they observe that the informed investor has sold shares, which moves the price downward. The informed investor can cover his short by buying from the active traders, or he can wait until after the further drop in price to cover, depending on how costly it is to carry

the short position another period. However, I do not make any special assumptions regarding the relative price elasticities of buys and sells. I also do not assume forced buying or selling by any class of traders. I assume that uninformed traders are willing to buy more shares at lower prices than those currently prevailing. Trade-based short sale manipulation is sustainable in a market setting in which due to information asymmetries, it is unclear whether the seller has negative information about the firm's prospects or is simply trying to manipulate the firm's stock price.

Naked short selling can increase the manipulator's profit. A short seller, who profits by buying the shares to cover her short position at lower prices than the selling prices, can drive the price of a stock lower by selling short a larger number of shares. Without enforceable restrictions requiring short sellers to borrow the shares before they can commit to sell, a short seller might destabilize the market for a particular stock through naked shorting.[9] While some naked shorting may take place for benign reasons, for example because it lowers the cost of short selling (Evans, Geczy, Musto, and Reed, 2003), Regulation SHO reflects the SEC's concern that previous restrictions on short selling had not been effective in preventing its use as a manipulative device (SEC, 2003b, 2004).[10] There is mounting evidence that manipulative short selling has seriously disrupted the market for some over-the-counter stocks.[11]

[9] Naked shorting creates so-called phantom shares, which give rise to a potential corporate governance problem. The buyer of the phantom shares usually does not realize they are not real shares and believes she has the same voting rights as the holders of real shares. Her broker will record the shares as a long position in her account and as a fail to receive on its books. If brokers send the proxy materials to owners of phantom shares, who then vote them, there could be more votes cast for directors than actually exist. See Curry. The SEC's proposed Regulation SHO (SEC, 2003) is designed to address the problem of naked short selling. In June 2004, the SEC announced a pilot program that would allow unrestricted short sales of 1,000 actively traded stocks for one year. At the same time, it announced a proposal to require broker-dealers to locate shares available for borrowing before engaging in any short sale. This rule was designed to curb naked short selling. "SEC Is Set to Approve Plan to Ease Short-Selling Curbs for One Year," *Wall Street Journal* (June 23, 2004).

[10] House Report (1991) expresses the same concern. The SEC recently adopted Regulation SHO to curb abusive short selling (SEC, 2003, 2004).

[11] Securities and Exchange Commission v. Rhino Advisors, Inc. and Thomas Badian, United States District Court, Southern District of New York, February 26, 2003, describes the naked short sale manipulation of the common stock of Sedona Corporation.

The unraveling problem should impose a constraint on naked shorting. There are two mechanisms for avoiding this constraint. Since a firm's common stock claims are extinguished if it liquidates, a manipulative short seller can effectively cover its short position at zero cost by forcing the firm into liquidation (House Report, 1991). Second, a popular private equity financing instrument, floating-price convertible securities (Hillion and Vermaelen, 2004), can resolve the unraveling problem because the manipulator does not have to buy back shares in the open market. He can obtain as many conversion shares as he needs by short selling the price downward just prior to the conversion notice date. The flawed structure of the floating-price convertible's contract may actually give security holders an incentive to manipulate the issuer's share price downward.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes the market equilibrium when there are no manipulators. Section 3 describes the market equilibrium when manipulators can enter the market. I assess the impact of short sale manipulation by comparing the two equilibriums. Section 4 explains how naked short selling can destabilize the market for a stock. Section 5 shows how floating-price convertibles resolve the unraveling problem, so that even trade-based short sale manipulation is profitable. Section 6 concludes.

2. The Market Model

This section characterizes the market equilibrium when there are no manipulators.

2.1 Institutional Details on Short Selling

A short sale is the sale of stock that the seller does not own.[12] The seller borrows the shares from a broker-dealer or an institutional investor. She establishes the short position by selling the borrowed shares and closes it out by buying the stock at a later date and returning the shares to the stock lender to extinguish the loan. Short sales increase the number of shares that are beneficially owned by investors and hence the stock's float.[13] As a result, the total number of shares beneficially owned and eligible to vote exceeds the number of shares the firm has issued.[14]

Short sales are heavily regulated in the United States both because of the riskiness of the strategy and also because of its potential for abuse as a manipulative device.[15] In the United States, many institutional investors are either prohibited by policy or regulation from short selling or tightly restricted as to the size of the short positions they can maintain. Many broker-dealers severely restrict short selling by their retail customers. However, the SEC has expressed concern that enforcement of the restrictions on short selling, and especially naked short selling, appears lax due to broker-dealers' tolerance of extended fails to deliver (SEC, 2003b, Boni, 2004).

The regulation of short selling in the United States has evolved from the recognition that unrestricted short selling could impair market efficiency by causing the price of a stock to spiral downward (Dechow et al., 2001). Regulation constrains short selling in several ways. SEC Rule

[12] Asquith and Meulbroek (1996), and Dechow, Hutton, Meulbroek, and Sloan (2001) describe the institutional arrangements of short selling in great detail. D'Avolio (2002) and Geczy, Musto, and Reed (2002) describe the market for stock loans. I provide just a brief summary.

[13] A common stock's float is equal to the number of outstanding shares minus the number of insider shares plus the short position in the stock.

[14] This has potentially significant corporate governance implications, which are beyond the scope of this paper (House Report, 1991, and SEC, 2003). The process of nominal share expansion through short selling and stock lending is very similar to the process of money supply expansion through bank lending, except that there is no 'reserve requirement,' only the clearing firm's willingness to arrange stock loans to cover the fails to deliver so that it can clear the trades, to control it.

[15] Because of these concerns, short selling is severely restricted in many foreign stock markets. Japanese securities regulators introduced a rule in February 2002 forbidding short sales at or below the current market price (Lilico, 2002). Taiwan regulations prohibit short selling by foreigners. All short selling in Hong Kong must be declared, and failure to do so is punishable by imprisonment.

10a-1 permits investors to sell short stocks listed on a national securities exchange only on either a "plus tick" or a "zero plus tick" (SEC, 2003)[16] The NASD has a similar bid test under NASD Rule 3350 but it only applies to Nasdaq National Market (NNM) securities when the trades are executed on either SuperMontage or over the NASD's Alternative Display Facility (ADF). The bid test does not apply to Nasdaq SmallCap, OTC Bulletin Board, or other over-the-counter stocks or to NNM securities traded away from SuperMontage or ADF unless the market in which they are traded has adopted its own price test. The short seller must place the proceeds from the short sale in an escrow account, which collateralizes the stock loan. The short seller can not use the short sales proceeds to hedge the short position. The short seller receives interest from the stock lender at a below-market interest rate, called the *rebate rate*, with the difference between the market rate and the rebate rate, the *rebate spread*, compensating the lender.[17] Federal Reserve Regulation T requires short sellers to post additional collateral in a margin account when the stock is shorted. The initial margin requirement is 50% of the market value of the shorted shares. The maintenance margin requirement is 25%.[18] Broker-dealers often set higher margin requirements, and large broker-dealers typically require at least 30% equity. The short seller has to top up the escrow account if the price of the stock rises but can reduce it if the price of the stock falls.

Current regulations prohibit naked short sales except under limited circumstances. New York Stock Exchange (NYSE) Rule 440c and NYSE Information Memorandum 91-41 (1991) require a short seller to make an affirmative determination that it will be able to borrow shares

[16] A "plus tick" occurs when the last trade occurred at a price higher than the last previous trade. A "zero plus tick" occurs when the last trade occurred at a price equal to the price of the last previous trade *and* the last prior trade that took place at a different price occurred at a higher price.

[17] The stock lending market is not a well-functioning competitive market (Ofek, Richardson, and Whitelaw, 2003). It is more appropriate to treat the rebate spread as an indicator of how difficult it is to borrow a stock, rather than as a competitively determined borrowing rate. Even though it is not a market price, it can still serve in the model as a useful proxy for the cost of borrowing stock.

[18] The stock exchanges and the NASD set the minimum maintenance margin requirements for their members. NYSE Rule 431 sets a 25% minimum for NYSE members.

before it can make a short sale unless the short seller is a market maker, specialist, or odd-lot broker who is selling short in connection with its normal market-making responsibilities. National Association of Securities Dealers (NASD) Rule 3370, NASD Rules of Fair Practice, Article III, Section 1, and SEC Release No. 34-35207 (1995) impose a similar affirmative determination requirement for NASDAQ stocks, and SEC Release No. 34-37773 (1996) imposes a similar requirement for American Stock Exchange-listed stocks.

The SEC recently adopted Regulation SHO to curb abusive short selling (SEC, 2003b, 2004). Rule 203 under Regulation SHO, which became effective January 3, 2005, prohibits a broker-dealer from accepting a short sale order unless it has arranged to borrow the security or has reasonable grounds to believe that it will be able to borrow it before the settlement date. It also requires the broker-dealer to enter into a bona-fide borrowing arrangement before executing an order to short sell any equity security that has been identified as a 'threshold security'. The threshold list includes any equity security that is either exchange-traded or is issued by a public reporting company for which aggregate fails to deliver at a registered clearing house amount to (a) at least 10,000 shares which represent (b) at least one-half of one percent of the issuer's outstanding shares.[19] It also requires the clearing house member or the clearing house to take action to cure all fails to deliver threshold stocks that persist for 10 days after the normal settlement date. The SEC proposed Regulation SHO out of concern that the existing rules restricting naked shorting had not been effective in preventing abuses (SEC, 2003b). However, the existing affirmative determination rules and the new rules under Regulation SHO except short sales executed by specialists and market-makers engaged in bona-fide market-making

[19] There are firms whose shares are quoted in the Pink Sheets but which are not subject to the public reporting requirements of the Securities Exchange Act of 1934. Such stocks are not covered by Regulation SHO.

activities (SEC, 2003b, 2004), which provides a potential loophole.[20] Boni (2004) finds that naked short sales are pervasive in the U.S. stock market, which supports the SEC's concern that broker-dealers have not been diligent in enforcing the existing short sale restrictions.[21]

Borrowing shares is costly. In addition to the cost implicit in receiving a below-market rebate rate, stock loan agreements typically require the borrower to reimburse the lender in full for any dividends or other distributions the issuer makes to its stockholders, which imposes a real cost (Frank and Jagannathan, 1998). Third, the Internal Revenue Code taxes all profits from short sales at the short-term capital gains rate, regardless of the length of time the position is open. Fourth, stock borrowers are exposed to the risk of a squeeze.[22]

Stock loan agreements usually provide that the loan must be repaid on demand. A short squeeze can occur when the lender demands the return of the shares but the borrower can not find a substitute lender and must therefore repurchase the shares in the open market. If the stock is thinly traded, or if there are a relatively large number of short sellers trying to cover their short positions, the resulting demand for shares can force the price higher and impose an added cost on short sellers. A short seller can mitigate this risk by borrowing on a term basis. However, term stock loans are unusual, and they are expensive (Geczy et al., 2002). Instead, market participants may use strategic fails to deliver (i.e., naked shorting) when stock borrowing is costly or impossible (Evans et al., 2003, Boni, 2004).

2.2 The Model

[20] The affirmative determination rules *do* apply even to market-makers when a stock has settlement failures that exceed the greater of (a) 0.5% of the stock's float and (b) 10,000 shares.

[21] Boni (2004) finds that 42% of listed stocks and 47% of unlisted stocks had fails of five days or more, and about 4% of the stocks had fails that would have classified them as 'threshold securities' under Regulation SHO. However, the median fails as a percentage of the outstanding shares was only 0.01% for NYSE, AMEX, and Nasdaq stocks and only 0.03% for OTCBB and Pink Sheet stocks. Both distributions are skewed because the mean fails as a percentage of outstanding shares was 0.19% for NYSE, AMEX, and Nasdaq stocks and 1.56% for OTCBB and Pink Sheet stocks.

[22] Dechow et al. (2001) cite as an example a short squeeze in the shares of Amazon.com in June 1998. Ofek and Richardson (2003) provide empirical evidence that rebate rates for Internet stocks were very high during the DotCom bubble, which implies a limited supply of shares available for loan and a relatively high risk of a squeeze.

The model is a simplified depiction of an actual stock market that still is able to capture the essence of manipulative short selling in actual stock markets. The model also gains considerable clarity without losing generality by assuming a non-dividend-paying stock and a zero interest rate. I assume that the intrinsic value of the stock to be revealed in the future can have either of two possible values, high (H) or low (L). I also assume that aside from the initial shareholders, stock market investors are of four types.[23]

First, there is an informed investor (subscripted I) who possesses information about the firm that enables him to know what the value of the stock will be when it is revealed to the market in the future. The informed investor could be a hedge fund or some other sophisticated investor. Insiders are also informed but are prohibited from short selling by corporate restrictions and the Securities Exchange Act of 1934.[24] One could also think of the informed investor as a professional short seller who has reliable information about the firm's future business prospects, which he gained through research (Diamond and Verrecchia, 1987). To simplify the model, I assume a single informed investor.

Because of the risks and the cost involved, short sellers are likely to be better informed than holders of long positions about the prospects for a stock (Diamond and Verrecchia, 1987). A short sale is the most direct way for an investor to bet that a stock's price will fall.[25] Short sellers expect the share price to fall sufficiently to compensate them for their costs and risks. Asquith and Meulbroek (1996) furnish empirical evidence that supports Diamond and Verrecchia (1987). They find a strong negative relation between the amount of short interest and subsequent stock returns, during both the period the stocks are shorted and the following two

[23] Aggarwal and Wu (2002) assume a similar market structure to model manipulative purchases.
[24] I assume that corporate blackout periods and the insider trading laws prohibit them from buying if they believe the firm's stock is undervalued. Thus, they do not buy shares to counter the short seller's manipulation. A more general model could allow for this behavior.
[25] According to Asquith and Meulbroek (1996), hedge fund managers and other professional investors have found that the option market is more expensive than short selling, especially for stocks that are hard to borrow.

years. They also find that those stocks that are heavily shorted for more than one month have the most negative returns.

Second, at times there is a manipulator (subscripted M), who I assume can also determine the stock's intrinsic value either through research or by observing the trading behavior of the informed investor. To simplify the model, I assume a single manipulator. The manipulator takes actions that are designed to drive down a stock's price, hoping to profit from the lower future price. The manipulator is capable of mimicking the informed investor, for example, by duplicating his volume of short sales, so as to conceal his manipulative intent from active traders and uninformed traders.

Manipulative strategies are of two general types. My model focuses on trade-based manipulation (Allen and Gale, 1992). The manipulator sells shares to drive down the price and hopes to profit by buying them back at lower prices in the future. Second, the manipulator could also engage in information-based manipulation by spreading rumors (Allen and Gale, 1992), engage in wash sales, or employ other manipulative devices without actually selling any shares to drive the price down. Such behavior violates Rule 10b-5 under the Securities Exchange Act of 1934 but it probably accounts for a significant portion of stock manipulation. The two strategies are complementary. By spreading false negative information after establishing the short position, a manipulator can further depress a stock's price and increase her profit. Reducing the price further gives the manipulator greater opportunity to cover her short position without driving the price up so much that it eliminates her profit. These non-trading devices could also be used to resolve the unraveling problem. In my model, the existence of active traders and the variable price feature of floating-price convertibles can both resolve the unraveling problem.

The manipulator can behave like an informed investor and as a manipulator at different times. He could act like an informed investor by selling short in anticipation of the stock's price

falling to L. He can also act like a manipulator by selling short to drive down the price and covering his short position before the share price is revealed to be H. In addition, in Section 4, I allow for the possibility that the manipulator can switch modes of behavior, at times borrowing shares to make routine short sales and at other times intentionally effecting naked short sales by failing to make delivery. Alternating between these two modes of behavior to exploit his information asymmetry disguises the manipulator's behavior and makes it more difficult for the regulators to detect his misbehavior and for the other market participants to interpret the signals in his trading decisions.

Third, there are N active traders (subscripted A_n , n = 1,….,N). Active traders, who may include market makers, search for information about whether the firm's stock price will be high or low in the future.[26] As part of their information gathering, they monitor the behavior of other traders to look for value signals. They observe market price and trading volume but they do not know the identities of buyers and sellers, which makes them incapable of distinguishing perfectly between sales by a manipulator and an informed investor.[27] They do not have complete information about the firm. Instead, they infer information from prices, trading volumes, and the trading behavior they observe in the market to decide whether they should buy the stock or sell it. They interpret sales by an informed investor (or by a manipulator they mistake for an informed investor) as a negative signal and sell shares the following period in response to the negative signal.

Fourth, there is a continuum of uninformed (or noise) traders (subscripted U). They initially have negligible holdings of the stock and behave like price takers. They do not

[26] Market makers may also be informed investors, depending on their access to information about the firm, or manipulators, depending on their trading motivation. I explain later in the paper that the manipulator has an incentive to register as a market-maker because of the exceptions to the short sale restrictions that apply to market-makers (but only to the extent of bona-fide market-making activities).

[27] It is certainly possible, of course, that the manipulator is also an insider. However, this is less likely when the insiders have large stock ownership because the manipulative short selling would also decrease the value of their shares.

condition their purchases on any specific information but instead, stand ready to buy more shares at lower prices, which provides liquidity to sellers. The uninformed traders' willingness to hold Q shares at time t is summarized in the following demand curve:

$$P = D(Q) = A - BQ, \quad H>A>L \geq 0, \quad B>0 \tag{1}$$

P is the market price of the stock at time t, and A and B are constants. The demand curve for the stock is downward-sloping (Shleifer, 1986, Kaul, Mehrotra, and Morck, 2000, and Liu, 2000).

At time 0, the firm's shares are held by insiders and passive investors who view their shareholdings as long-term investments. A portion of the firm's shares are held in margin accounts with broker-dealers where they are available for lending to short sellers.[28] If no one wishes to sell the stock, then its price is A. The total number of shares outstanding is (A – L)/B. If the time zero shareholders wish to sell all the outstanding shares to uninformed traders, then the price would fall to L.

Share transactions occur in the market in the following sequence. At time 1, either the informed investor or the manipulator can initiate a short sale. Since neither the informed investor nor the manipulator owns any shares, each must borrow them. I relax this assumption later when I consider the possibility of naked short sales. The informed investor sells shares if and only if the future stock price will be L. The probability that the future stock price will be L is p (and the probability that it will be H is 1 – p). One can think of A, the current market price, as the expected present value of the share price at time 3:

$$A = pL + (1-p)H \tag{2}$$

[28] Shares held in cash accounts are not available for lending without the account holder's permission. Shares held in margin accounts are freely lendable. I assume that the margin account holders are uninformed investors. Alternatively, it could be assumed that a portion of the shares are held by a fifth class of shareholders, passive investors, such as stock index funds or mutual funds, who intend to hold them for the long term and are willing to lend them to short sellers in order to earn extra income in the form of stock loan rebates.

The manipulator observes the informed investor's trading. She will not sell the stock short if the informed investor does, and she may decide not to enter the market even if the informed investor is not selling.[29] The manipulator sells shares with probability $q < 1 - p$.[30] There is a probability $1 - p - q$ that neither the informed investor nor the manipulator will engage in short selling.

Active traders observe the stock price and trading volume at time 1. They sell shares at time 2 based on what they learn at time 1 conditioning their decision to sell on whether they observe an informed investor (or the manipulator whom she mistakes for the informed investor) selling.[31] The manipulator or the informed investor can buy or sell shares at time 2. The uninformed traders stand ready to buy shares at time 1 and also at time 2. The stock's value is revealed to be H or L per share at time 3.

The informed investor or the manipulator can sustain a short position until time 3 but it is less expensive to sustain it to time 2 (unless the manipulator naked shorts). One might think of this assumption in any of three ways. First, the rebate spread represents a direct cost of carrying the short position. This cost can exceed the market rate of interest when the stock is on special and extremely hard to borrow (D'Avolio, 2002, Duffie, Garleanu, and Pedersen, 2002, Geczy, Musto, and Reed, 2002). Second, time 3 represents the long run, and it may be very costly for the informed investor or the manipulator to borrow the shares to maintain the short position, for example, because she is unable to borrow the stock continuously over an extended period.[32]

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[29] In a market equilibrium in which the informed investor sells the profit-maximizing number of shares, I show later in the paper that incremental short sales by the manipulator will not be profitable.

[30] Later in the paper I determine the optimal probability of manipulation and show that if the probability of manipulation is too high, then the active traders refuse to sell shares and the manipulative scheme fails.

[31] If there are no stock sales at time 1, then it is reasonable to assume that active traders will purchase shares at time 2 until they raise the price to H. I do not address this possibility in my model because my focus is on what happens when there are short sales at time 1.

[32] A stock lender can get the shares back on demand. In that case, the short seller's broker must try to borrow replacement shares from some other shareholder to keep the short position open. If the broker can not borrow the shares, then it must close out the short position.

Third, while there is no uncertainty in my model, I could motivate a cost to maintaining the short position that risk-averse investors face by making the distribution of time 3 prices uncertain. Instead, I model the cost of holding the short position until time 2 as a scalar C per share and to time 3 as 2C per share. D'Avolio (2002) finds that the overall value-weighted cost to borrow stocks is 25 bp per year; 91% of the stocks ("general collateral" stocks) cost less than 1% per year to borrow with a mean-weighted fee of only 17 bp; but the other 9% ("special" stocks) have a mean fee of 4.3% per year; and less than 1% ("extremely special" stocks) have negative rebate rates as high as 50%. If the stock price at time 3 is L, then the informed investor's cost of shorting a share until time 3 is L + 2C. Unless A – L – 2C > 0, the informed investor would never sell shares at a price less than or equal to the time 0 price and maintain the short position until time 3. To simplify the model, I also assume that active traders incur at most a negligible cost to holding a short position.[33]

2.3 Market Equilibrium

I investigate the impact of short sale manipulation on stock market equilibrium by comparing two market settings. In the first, there is an informed investor and active traders but no manipulator. Both can sell shares short. They sell short when they expect the equilibrium price of the shares to drop to L at time 3. In the next section, I permit a manipulative short seller to enter the market and examine how her trading alters the market equilibrium. Unlike legitimate short sellers, the manipulators sell short in the hope that their selling drives the share price below the shares' intrinsic value and attracts other sellers from whom they can buy shares after the price drop to cover their short positions.

[33] Market makers have lower shorting costs than other market participants because they can sell on a downtick and also because they do not have to make an affirmative determination that they will be able to borrow shares before they sell short. Market makers are granted these exceptions to facilitate their market-making activities. A strategy a manipulator can employ to reduce its cost of shorting is to register as a market maker for the target stock (SEC, 2003). With the assumption of zero cost for active trader shorting, C can be thought of as the incremental shorting cost the informed investor and the manipulator must pay as compared to active traders. Later in the paper I consider naked shorting, which I assume to have zero cost.

The informed investor might sell $Q_I(1)$ shares short at time 1 that he plans to repurchase at time 3, an additional $\hat{Q}_I(1)$ shares short at time 1 that he plans to repurchase at time 2, and a further $Q_I(2)$ shares short at time 2, which he would repurchase at time 3.[34] I show that when shorting is expensive, the optimal strategy for the informed investor is to sell shares at time 1 but neither to buy nor to sell shares at time 2. When shorting is inexpensive, the informed investor will sell shares short in both periods. Initially, I assume that there are N symmetric active traders but no manipulator. Each active trader sells $Q_A^i(2)$ shares short to the uninformed investors at time 2 if she observes what she believes to be the informed investor selling at time 1.[35] There are no limits on the number of shares short sellers can borrow.[36]

The active traders believe that the informed investor has negative information about the firm's prospects when they have observed him selling at time 1. Each active trader realizes that she is competing against N – 1 other active traders to sell her shares. The aggregate number of shares the active traders offer for sale is:

$$Q_A(2) = \sum_{i \in N} Q_A^i(2) \qquad (3)$$

where $Q_A^i(2)$ is active trader i's offer to sell at time 2. All the outstanding shares at time 2 are available for sale because the uninformed investors can sell the $Q_I(1) = Q_U(1)$ shares they purchased from the informed investor at time 1.[37]

Each active trader solves the following problem:

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[34] I assume that the informed investor does not buy shares when he realizes that the time 3 price will be H either because his charter limits him to short selling or because he believes he can find other short selling opportunities that are more profitable.

[35] I develop this case further when I introduce a manipulator into the market. To distinguish her behavior from that of the manipulator, suppose he has no shares. I assume that the informed investor in that case would want to release any credible negative information he has concerning the true value of the shares into the market before he buys any shares. Since he is not a manipulator, I assume that any information he releases is credible.

[36] Later in the paper I consider the impact of a limitation on the number of shares that are available for short sellers to borrow.

[37] As long as there is at least one active trader, the aggregate number of shares offered by active traders at time 2 will exceed the number of shares demanded by the informed investor, $Q_A(2) > Q_I(2)$.

$$\max_{Q_A^i(2)}(A - B[Q_I(1) + Q_I(2) + \sum_{i \in N} Q_A^i(2)])Q_A^i(2) - LQ_A^i(2) \tag{4}$$

subject to $Q_A^i(2) \geq 0$. Solving the N first order conditions gives

$$Q_A^*(2) = \frac{A - L - BQ_I(1) - BQ_I(2)}{B(N+1)} \tag{5}$$

Given the assumed symmetry of the active traders, equation (5) holds for each. The equilibrium market price is

$$P^*(2) = \frac{A + NL - BQ_I(1) - BQ_I(2)}{N+1} \tag{6}$$

The informed investor sells shares at time 1 and repurchases them either at time 2 or at time 3. He decides how many shares to sell at time 1 by solving the following problem:

$$\max_{Q_I(1), \hat{Q}_I(1)}(A - B[Q_I(1)) + \hat{Q}_I(1)])(Q_I(1) + \hat{Q}_I(1)) - (L + 2C)Q_I(1) - (P^*(2) + C)\hat{Q}_I(1) \tag{7}$$

subject to $Q_I(1) \geq 0$ and $\hat{Q}_I(1) \geq 0$. Applying the Kuhn-Tucker conditions, equation (7) has the following solutions. Either $Q_I(1) = 0$ or

$$Q_I(1) = \frac{A - L - 2C}{2B} - \frac{2N+1}{2N+2}\hat{Q}_I(1) \tag{8}$$

Either $\hat{Q}_I(1) = 0$ or

$$\hat{Q}_I(1) = \frac{A - P^*(2) - C}{2B} - Q_I(1) \tag{9}$$

$Q_I(1) \geq 0$ provided $C \leq (3N+3)(A-L)/(2N^2 + 5N + 5)$, and $\hat{Q}_I(1) \geq 0$ provided $C \geq (A-L)/(2N)$.

The informed investor's strategy at time 2 must be optimal given the N active traders' demand for shares at that time. The informed investor solves the problem:

$$\max_{Q_I(2)}(A - B[Q_I(1) + Q_I(2) + \sum_{i \in N} Q_A^i(2)])Q_I(2) - (L + C)Q_I(2) \tag{10}$$

subject to $Q_I(2) \geq 0$. The solution to equation (10) is $Q_I(2) = 0$ or

$$Q_I(2) = \frac{A - L - BQ_I(1) - (N+1)C}{B(N+2)} \tag{11}$$

$Q_I(2) \geq 0$ provided $C \leq (A-L)/(2N)$.

2.4 Equilibrium When Selling Short Is Expensive

When the informed investor's cost of shorting shares is high enough and the number of active traders is large enough that $C \geq (A-L)/(2N)$, then $Q_I(1) = 0$, $Q_I(2) = 0$, and $\hat{Q}_I(1) > 0$. The informed investor only sells shares short at time 1 and repurchases at time 2 all the shares he shorted at time 1. Each active trader sells short

$$\mathrm{Q}_{\mathrm{A}}^{*}(2) = \frac{\mathrm{A} - \mathrm{L}}{\mathrm{B}(\mathrm{N}+1)} \tag{12}$$

shares. The aggregate number of shares the N active traders offer to sell at time 2 is:

$$\overline{\mathrm{Q}}_{\mathrm{A}}^{*}(2) = \frac{\mathrm{N}}{\mathrm{N}+1}\frac{\mathrm{A} - \mathrm{L}}{\mathrm{B}} \tag{13}$$

The time 2 price is:

$$P^{*}(2) = A - B\overline{Q}_A^{*}(2) = \frac{NL + A}{N+1} = L + \frac{A-L}{N+1} \tag{14}$$

Each active trader expects to earn profit of

$$\pi^{A_i} = \frac{(A-L)^2}{B(N+1)^2} \tag{15}$$

Table 1 shows how the market equilibrium depends on the cost of shorting and the number of active traders.

As the number of active traders becomes large, the aggregate short position converges to all the outstanding shares, and $\mathrm{P}^{*}(2)$ converges to the shares' intrinsic value:

$$\lim_{N\to\infty} \bar{Q}_A^*(2) = \frac{A-L}{B} \tag{16}$$

$$\lim_{N\to\infty} P^*(2) = L \tag{17}$$

Competition among active traders promotes market efficiency. However, if there is only a small number of active traders, competition is limited, and each will try to extract a per-share rent equal to

$$P^*(2) - L = (A-L)/(N+1) \geq C \tag{18}$$

The informed investor sells

$$\hat{Q}_I^*(1) = \frac{A - P^*(2) - C}{2B} = \frac{N(A-L) - (N+1)C}{2B(N+1)} \tag{19}$$

shares at time 1 at a market price of

$$P^*(1) = A - \frac{N}{N+1}\frac{A-L}{2} + \frac{C}{2} = L + \frac{N+2}{2N+2}(A-L) + \frac{C}{2} \tag{20}$$

She will not find it profitable to sell shares short at time 2 nor to maintain the short position until time 3. With this strategy, the informed investor realizes a profit equal to

$$\pi_I^* = \frac{[N(A-L) - (N+1)C]^2}{4B(N+1)^2} \tag{21}$$

The informed investor will not find it profitable to sell shares at time 2. If he were going to sell additional shares, he would be better off selling them at time 1 because $P^*(1) > P^*(2)$. The informed investor also will not buy any shares at time 2 beyond what would be required to cover his short position because he would lose $P^*(2) - L$ on each net share he purchased at time 2 and held to time 3.[38] He will not sell any shares at time 2 because he would lose $L + C - P^*(2)$ on each share he sold short at time 2 and held to time 3. He will not hold his short position to

[38] Unless the holding period exceeds six months, Rule 16b under the Securities Exchange Act of 1934 would obligate any 10 percent shareholder, officer, or director to return to the firm the so-called short swing trading profits earned from selling and repurchasing the stock within a six-month period.

time 3 because the cost of holding it and closing it out at time 3 is L + 2C. Since $C \geq (A-L)/(2N),$ this strategy is less profitable than repurchasing the shares at time 2 because in that case his profit would only be:

$$\pi_I = P^*(1)\hat{Q}_I^*(1) - (L+2C)\hat{Q}_I^*(1)$$

$$= \frac{[N(A-L)-(N+1)C]^2}{4B(N+1)^2} + \frac{[A-L-(N+1)C][N(A-L)-(N+1)C]}{2B(N+1)^2} \leq \pi_I^* \qquad (22)$$

Finally, each active trader's strategy is optimal given all the other active traders' strategies and the informed investor's strategy. Thus, no active trader can deviate profitably.

2.5 Equilibrium When Selling Short Is Inexpensive

When the informed investor's cost of maintaining the short position is low enough and the number of active traders is small enough that $C \leq (3N+3)(A-L)/(2N^2+5N+5),$ then $Q_I(1) \geq 0$, $Q_I(2) \geq 0$, and $\hat{Q}_I(1) = 0.$ The informed investor sells additional shares short at time 2 and waits until time 3 to cover his entire short position.[39]

The informed investor sells more shares short at time 1 when the cost of shorting is low because

$$Q_I(1) + \hat{Q}_I(1) = \frac{A-L-2C}{2B} > \frac{N(A-L)-(N+1)C}{2B(N+1)} \qquad (23)$$

The active traders sell fewer shares short at time 2 because

$$\overline{Q}_A^*(2) = \frac{A-L+4C}{2B}\frac{N}{N+2} < \frac{N}{N+1}\frac{A-L}{B} \qquad (24)$$

provided N > 1. But the total number of shares shorted at time 2 is greater, and as a result, $P^*(2)$ is lower than in the high-shorting-cost case:

[39] When $(A-L)/(2N) < C < (3N+3)(A-L)/(2N^2+5N+5)$, the informed investor repurchases at time 2 a portion of the shares initially sold short and the rest at time 3 but will not sell short any additional shares at time 2.

$$P^*(2) = L + \frac{A - L + 4C}{2(N+2)} < L + \frac{A - L}{N+1} \tag{25}$$

provided N > 1.

$P^*(1)$ is also lower, and therefore, closer to the shares' intrinsic value, because

$$P^*(1) = \frac{A+L}{2} + C < A - \frac{N}{N+1}\frac{A-L}{2} + \frac{C}{2} \tag{26}$$

provided N > 1 due to the heavier short selling by the informed investor at time 1 when the cost of shorting is lower. Less expensive short selling facilitates arbitrage and promotes market efficiency.

Also, as in the high-cost case, the aggregate short interest converges to all the shares outstanding and $P^*(2)$ converges to the shares' intrinsic value as the number of active traders becomes large:

$$\lim_{N\to\infty} Q_I(1) + Q_I(2) + \overline{Q}^*_A(2) = \frac{A-L}{B} \tag{27}$$

$$\lim_{N\to\infty} P^*(2) = L \tag{28}$$

Thus, in both cases, competition among active traders promotes market efficiency.

2.6 Timing of Short Covering

The informed investor will hold the short position until time 3, rather than cover it at time 2, provided

$$L + C < P^*(2) = L + \frac{A - L + 2C}{2(N+1)} \tag{29}$$

which is satisfied when $C < (A-L)/(2N)$. So long as C is small or N is small, the informed investor finds it more profitable to cover the short position at time 3. He also sells short more shares at time 2 and covers those short sales at time 3. But as the number of active traders grows, eventually the sign in equation (29) reverses. The informed investor stops selling shares short at

time 2, and he repurchases some at time 2 and the rest at time 3. When N grows large enough that $C > (A - L)/(2N)$, the informed investor only sells shares short at time 1 and covers the entire short position at time 2.

Active traders have two opposing effects on the informed investor's profit. First, they sell shares at time 2, which reduces P*(2) and the informed investor's information rent from short selling, P*(2) – L. The lower P*(2) allows the informed investors to repurchase shares more cheaply at time 2 in the expensive-shorting case (IV). As a result, they sell more shares $\hat{Q}_I^*(1)$. Increasing the number of active traders in the expensive-shorting case increases π_I^*.[40] On the other hand, the active traders compete with the informed investor to sell shares short at time 2 when short selling is inexpensive enough (and the number of active traders is small enough) that the informed investor wants to sell shares short at time 2 (case II). Greater competition reduces the informed investor's information rent. This reduces $Q_I(2)$ and the profitability of the informed investor's short selling in the low-shorting-cost case (II), since π_I^* in the low-shorting-cost case depends on N:

$$\pi_I^* = \frac{(A - L - 2C)^2}{4B} + \frac{(A - L - 2NC)^2}{4B(N+2)^2} \tag{30}$$

If the number of active traders becomes large enough – and the competition becomes sufficiently intense – the informed investor's profit-maximizing strategy shifts from selling shares to buying them back at time 2. Short selling at time 2 becomes less profitable as N increases and P*(2) falls. Competition from more active traders eventually makes it unprofitable to sell short at time 2. He buys rather than sells at time 2. Further increases in the number of active traders raise $\hat{Q}_I(1)$ and increase the informed investor's profit. This equilibrium

[40] π_I^* increases with N in case IV except when A – L is very small, which is not an interesting case because the profit potential in short selling is small.

corresponds to the high-shorting-cost case. The change in strategy in response to the increase in the number of active traders makes the informed investor worse off because even though he only incurs one period's shorting cost, he must pay P*(2) > L to repurchase the shares. His profit is greater in the low-shorting-cost case because π_I^* in equation (30) exceeds π_I^* in equation (21).[41]

2.7 Effect of Short Sale Restrictions

Suppose the number of shares available for borrowing is capped at H. Then

$$Q_I(1) + \hat{Q}_I(1) \le H \tag{31}$$

$$Q_I(1) + Q_I(2) + \overline{Q}_A(2) \le H \tag{32}$$

As a result, P*(2) in equation (6) includes a shadow price for short sales λ. The time 2 market price in equations (17) and (28) converges to L + λ. The amount of short sales at time 2 in equations (16) and (27) converges to H < (A – L)/B. The borrowing restriction reduces market efficiency by preventing short sellers from arbitraging away the mispricing (Dechow, Hutton, Meulbroek, and Sloan, 2001, D'Avolio, 2002, and Geczy, Musto, and Reed, 2002).

The market equilibrium in the simplified market structure exhibits the behavioral properties one would expect in a market that is free of manipulation.[42] Figure 1 illustrates the sensitivity of market prices to the cost of shorting and to the number of active traders, and Figure 2 illustrates the sensitivity of π_I^* to N.

3. Market Equilibrium When Manipulators Are Present in the Market

Next, I consider how a manipulator entering the market affects the market equilibrium. If the informed investor is shorting shares optimally and the manipulator has the same cost of

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[41] Since $C \le (A-L)/(2N)$, π_I^* in equation (30) is no less than $(N-1)^2(A-L)^2/(4BN^2)$. Since $C \ge (A-L)/(2N)$ for π_I^* in equation (21), in that case $\pi_I^* < (N-1)^2(A-L)^2/(4BN^2)$ for N > 0.

[42] Table 1 contains a third case, which might be termed the 'moderate cost of shorting case.' It is easily verified that the market equilibrium in this case also exhibits the expected behavioral properties.

shorting, then it will be unprofitable at the margin for the manipulator to sell shares short.[43] Therefore, I assume that the manipulator does not short shares if the informed investor is selling shares short.[44] If the informed investor does not sell any shares short at time 1, then the manipulator knows that the price will be H at time 3. In that event, the manipulator sells shares short with probability q/(1 – p). The active traders continue to condition their behavior at time 2 on what they observe at time 1. Market equilibrium can be either a pooling equilibrium or a separating equilibrium, depending on the cost of shorting.

3.1 Pooling Equilibrium

A pooling equilibrium can occur in case IV but not in the other three cases in Table 1. The manipulator can imitate the selling behavior of the informed investor by shorting $\hat{Q}_I^*(1)$ shares at time 1. The manipulator must cover his short position at time 2 because holding the shares until time 3 is unprofitable.

If the manipulator sells $\hat{Q}_I^*(1)$ shares at time 1, the same number as the informed investor, then the active traders will assess the likelihood that the seller is a manipulator as:

$$\beta = q/(p+q) \tag{33}$$

Each active trader solves the following problem at time 2 conditional on observing a sale at time 1:

$$\max_{Q_A(2)} (1-\beta)[(A - B\sum_{i\in N} Q_A^i(2))Q_A^i(2) - LQ_A^i(2)] + \beta[(A - B\sum_{i\in N} Q_A^i(2))Q_A^i(2) - HQ_A^i(2)] \tag{34}$$

The N active traders sell fewer shares short at time 2 as a result of the risk of manipulation:

$$\overline{Q}_A^*(2) = \frac{N}{N+1}\frac{A-(1-\beta)L-\beta H}{B} < \frac{N}{N+1}\frac{A-L}{B} \tag{35}$$

[43] Later in the paper I show that it is profitable for the manipulator to sell shares short if he has lower shorting costs than the informed investor. Naked shorting satisfies this condition.

[44] The manipulator could spread rumors or engage in manipulative trading to drive the informed investor from the market. I do not consider the implications of such behavior in this paper.

Consequently, the market price at time 2 is higher due to manipulation:

$$P_M^*(2) = A - B\overline{Q}_A^*(2) = L + \frac{A - L + N\beta(H - L)}{N + 1} > L + \frac{A - L}{N + 1} = P^*(2) \tag{36}$$

Each active trader expects to earn smaller profits than it would in a market that is free of manipulation:

$$\pi^{A_i} = \frac{[A - (1 - \beta)L - \beta H]^2}{B(N + 1)^2} < \frac{(A - L)^2}{B(N + 1)^2} \tag{37}$$

The adverse impact of the manipulation on the active traders' profits increases with the relative likelihood of manipulation (β).

If either the informed investor or the manipulator sells shares short, then each sells

$$\hat{Q}_I^*(1) = \hat{Q}_M^*(1) = \frac{N[A - (1 - \beta)L - \beta H] - (N + 1)C}{2B(N + 1)} \tag{38}$$

shares at a price of

$$P^*(1) = A - \frac{N[A - (1 - \beta)L - \beta H] - (N + 1)C}{2(N + 1)} = L + \frac{(N + 2)(A - L) + N\beta(H - L)}{2(N + 1)} + \frac{C}{2} \tag{39}$$

and buys them all back at time 2 at price $P_M^*(2)$. $P^*(1)$ is higher the greater is the perceived risk (β) that the market may be manipulated. The informed investor or the manipulator earns expected profit equal to

$$\pi_M^* = \pi_I^* = \frac{[N(A - L - \beta(H - L)) - (N + 1)C]^2}{4B(N + 1)^2}, \tag{40}$$

which is positive provided

$$\beta < \frac{N(A - L) - (N + 1)C}{N(H - L)} \tag{41}$$

The manipulator's profit is inversely related to the likelihood β the active traders attach to the possibility of manipulation, and thus, to how well the manipulators disguise their activity. But so

long as $\beta > 0$, π_M^* in equation (40) is less than π_I^* in equation (21). The informed investor (and the manipulator) earns lower expected profit from short selling when the market can be manipulated than the informed investor earns from short selling when the market is free of manipulation.

For the pooling equilibrium to be sustainable, the informed investor must not have an incentive to deviate from this strategy to separate himself from the manipulator. Selling a different number of shares at time 1 but still buying them back at time 2 will not disrupt the pooling equilibrium because it is costless for the manipulator to mimic this strategy. In addition, there is no credible way for the informed investor to commit to staying short through time 3 because the active traders only observe the quantity sold and the price at time 1.[45] Thus, the pooling equilibrium will be sustained so long as the informed investor finds it more profitable to buy back shares at time 2 rather than at time 3.

The cost to the informed investor of staying short until time 3 is L + 2C. So the incentive compatibility condition is satisfied when it is cheaper to buy back the shares at time 2: $P_M^*(2) < L + C$. Rearranging this condition gives

$$C > \frac{A - L + \beta N(H - L)}{N + 1} \tag{42}$$

or

$$\beta < \frac{C(N+1) + L - A}{N(H - L)} \tag{43}$$

The incentive compatibility condition is satisfied when the number of active traders is sufficiently large, the cost of shorting the stock is sufficiently high, and the relative likelihood of manipulation β is sufficiently small.

[45] If either the informed investor or the manipulator tries to release information, the pooling equilibrium can not be sustained unless the manipulator can appear as credible as the informed investor.

The greater the cost of staying short until time 3, the greater the likelihood the informed investor will cover his short position at time 2, and the easier it is for the manipulator to pool with the informed investor. Second, the greater the likelihood that the seller of shares at time 1 is a manipulator, the more severe is the adverse selection problem facing the active traders. They respond by selling fewer shares at time 2, which causes the price to be higher than it otherwise would and makes it less likely that the manipulator will be able to pool with the informed investor. Since β is inversely related to the probability the seller is an informed investor, it is easier for the manipulator to sustain the pooling equilibrium when the active traders believe it is more likely that the seller is an informed investor. Third, the right-hand side of inequality (43) is decreasing in H – L. The greater the dispersion of future share prices, the lower the likelihood of a pooling equilibrium. The greater the price dispersion, the more profitable it is for the informed investor to wait until time 3 to cover his short position.

Fourth, an increase in the number of active traders raises the likelihood of a pooling equilibrium because the right-hand side of inequality (43) is increasing in N. Increasing the number of active traders reduces the cost of covering the short positon at time 2 and makes it more likely that the incentive compatibility condition for a pooling equilibrium is satisfied. However, market efficiency suffers because less information concerning the market manipulation is revealed. This effect suggests an important role for government regulation. In the absence of manipulation, an increase in the number of active traders would improve market efficiency by driving $P^*(2)$ closer to the stock's true value. This effect is less pronounced when manipulators are present in the market. Inequality (43) suggests that decreasing the conditional probability β that a manipulator is present increases the likelihood that the manipulation will be concealed by a pooling equilibrium. However, equation (35) shows that reducing β improves market efficiency by driving $P_M^*(2)$ closer to the manipulation-free price $P^*(2)$. More effective

regulatory enforcement of the rules barring manipulation can improve market efficiency by reducing the likelihood that manipulators are present in the market. But it may also increase the likelihood that if manipulation occurs, it will be concealed by a pooling equilibrium.

3.2 Separating Equilibrium Free of Manipulation

A separating equilibrium can occur in cases I, II or III in Table 1. The manipulator would still want to cover his short position at time 2. First I consider case II. The informed investor sells shares short at time 1 and again at time 2 but waits until time 3 to cover. The manipulator will not find it profitable to sell short.

The active traders would sell

$$\overline{Q}_A^*(2) = \frac{N}{N+2}\frac{A-L-4\beta(H-L)+4C}{2B} < \frac{N}{N+2}\frac{A-L+4C}{2B} \tag{44}$$

shares, which is fewer shares than they would sell in a market that is manipulation-free. $P^*(1)$ is unaffected by the manipulation but the time 2 price is higher:

$$P_M^*(2) = L + \frac{A-L+2N\beta(H-L)+4C}{2(N+2)} > L + \frac{A-L+4C}{2(N+2)} \tag{45}$$

The manipulator would suffer a loss if he mimicked the informed investor by selling shares short at $P^*(1)$ and $P_M^*(2)$ and repurchasing them at a cost per share of H. If the manipulator tries to cover his time 1 short position at time 2 (buying rather than selling), the active traders would be alerted to the manipulation, the market price would adjust upward, and the manipulator would be precluded from earning a profit. The manipulator can not profitably mimic the informed investor's strategy if he only enters the market when the informed investor does not. But if he enters and so does the informed investor, then his (incremental to the market) short sales are unprofitable. Thus, a separating equilibrium occurs when the cost of shorting is sufficiently small and the number of active traders is sufficiently small.

A separating equilibrium can also occur in case III. The manipulator would have to sell $Q_I(1) + \hat{Q}_I(1)$ shares at time 1 to mimic the informed investor, and he would have to repurchase $\hat{Q}_I(1)$ shares at time 2 and the rest at time 3 to avoid detection. However, while he would make a profit on the shares he repurchases at time 2, he would suffer an even greater loss on those he repurchases at time 3. A sustainable equilibrium occurs only if the informed investor is the sole short seller in case III.[46]

Manipulative short selling can not occur in a separating equilibrium. Two conditions must be satisfied. First, the informed investor must be willing to wait until time 3 to cover her short position. If she is patient enough, then a potential manipulator will not be able to mimic her strategy profitably. This condition requires a relatively small cost of shorting. Second, if the number of active traders is small, then the informed investor's best strategy is to hold the short position until time 3.

3.3 Role of Active Traders

Active traders affect the market in opposite ways. If the number of active traders is small, increasing their number provides the usual benefit of arbitrage by incorporating the information embodied in the informed investor's sell signal into the market price to improve market efficiency. However, as the number of active traders becomes large, their presence eventually has the opposite effect. A pooling equilibrium can occur. Increasing the number of active traders depresses $P^*(2)$ to such an extent that the informed investor prefers to repurchase shares at time 2, rather than wait until time 3 to buy them. The manipulator is able to mimic the informed investor's strategy. As a result, $P^*(2)$ deviates farther from the shares' intrinsic value

[46] The manipulator can earn a profit if he sells $Q_I(1) + \hat{Q}_I(1)$ shares short at time 1 and buys them all back at time 2 provided the active traders' perceived likelihood of manipulation is sufficiently low. However, such an equilibrium is not sustainable. The active traders would observe the manipulator completely covering his short position at time 2 and the informed investors behaving differently, and they would refrain from trading when they observe the manipulators completely short covering at time 2.

than in the manipulation-free case because the active traders do not know whether the short seller at time 1 is an informed investor or a manipulator. Interestingly, increasing the number of active traders actually increases the likelihood of manipulation and impairs market efficiency because the informed investor is less willing to wait until time 3 to cover her short position. As a result, the manipulator can more easily camouflage his true intent.

An important implication of the separating equilibrium is that manipulation is less likely to affect stocks for which there is a large pool of shares available for lending because the cost of shorting is lower and a separating equilibrium is more likely.[47] Manipulation is more likely when the informed investor has a high cost of shorting (for example, when the stock is on broker-dealers' 'hard-to-borrow' lists) but the manipulator has a much lower cost, for example, zero cost through strategic fails to deliver.

The manipulator's inability to sell short profitably when borrowing shares is expensive would seem to suggest that if naked shorting were not permitted, then manipulative short selling would be less likely to affect stocks that are difficult (or impossible) to borrow, for example, because there is only a small public float or a large portion of the public float is in the hands of institutions or individuals who hold their shares in cash accounts and refuse to lend. Naked shorting avoids this constraint because it enables the manipulator to short even hard-to-borrow stocks.

4. Naked Short Selling and Strategic Fails to Deliver

The SEC recently adopted Regulation SHO "to address the problem of 'naked' short selling" (SEC, 2003, p. 1; 2004). Naked short selling involves entering into a transaction to sell

[47] A larger pool of available shares makes it less likely that the shares will be on special and also less likely that the borrower would face a premature demand for the return of the shares that would force him to close out the short position.

shares that the seller does not own and has not arranged to borrow to deliver to the buyer. A naked short sale may initially appear to the clearing house as a routine fail to deliver (on the settlement date). The naked short sale becomes apparent when the seller fails to deliver for an extended period.[48] Miller (1977) shows that the marginal investor is an optimist when short selling is constrained and investors' opinions on the stock diverge. Naked short selling removes the constraint on short selling and results in a pessimist/manipulator becoming the marginal investor. Naked short selling can be especially destabilizing to a company's share price when unrestrained because ignoring the regulatory requirement to borrow the shares eliminates the main quantitative constraint on the amount of short selling and intensifies the resulting downward pressure on price. There have been instances where the short position in a stock has exceeded the firm's entire supply of outstanding shares.[49]

Evans, Geczy, Musto, and Reed (2003) introduce the concept of strategic failures to deliver (or more simply, *strategic fails*), which occur when short sellers decide not to borrow shares and deliver them because borrowing the shares is too expensive or too difficult. They show that options market-makers use strategic fails to get around short selling constraints affecting stocks with listed options. Boni (2004) documents the prevalence of strategic fails -- naked short sales -- across the entire spectrum of U.S. stocks, including stocks that do not have options listed, and provides evidence that market-makers strategically fail to deliver shares whose borrowing costs are high. Manipulative short sellers have an incentive to use strategic

[48] The NASD has several rules that address failure to deliver. Rule 3210 prohibits an NASD member from selling a security for its own account or buying the security as a broker for a customer if it has a fail to deliver in that security for 60 days or longer. Rule 11830 imposes a mandatory close-out requirement when a broker-dealer's clearing short position in a stock is 10,000 or more shares and represents one-half of one percent or more of the issuer's total shares outstanding. The broker-dealer for the seller in that case must close out any short position for which the fail to deliver has persisted for 10 days or more beyond the normal settlement date. However, this mandatory close-out requirement does not apply to bona-fide market-making transactions or to transactions that result in fully hedged positions. Regulation SHO, which became effective January 3, 2005, requires the clearing house to close out any fail to deliver position in a threshold security that persists for 13 consecutive settlement days.

[49] Regulation SHO stemmed from the SEC's concern about the extent of naked shorting, including instances of massive naked shorting the SEC had detected in which the short position exceeded the number of known outstanding shares. See "New Rules to Put Squeeze on Shorts," *Wall Street Journal* (January 27, 2005): C5.

fails because they avoid having to pay the rebate spread. Evans et al. (2003) also provide evidence that broker-dealers seldom request buy-ins, which reduces the risk that a stock loan might be called prematurely and leave the borrower unable to arrange a replacement loan.[50]

Naked short selling and manipulating the price downward provide cash returns to the manipulator, who can withdraw cash from his clearing firm account as the shorted shares are marked to market at progressively lower prices.[51] Through naked shorting, the manipulator realizes these returns without investing any cash (provided the market price never rises above the sale price).

4.1 The Role of the Clearing House

Naked short selling could not occur, or at least not persist, if the stock purchaser or the clearing house insisted on taking delivery of the shares. Most common stock transactions in the United States clear through the National Securities Clearing Corporation (NSCC). The NSCC is a subsidiary of the Depository Trust and Clearing Corporation (DTCC). Another subsidiary of DTCC, the Depository Trust Company (DTC), is the world's largest securities depository and serves as the clearing house for most trades of registered shares in the United States.[52] DTC was formed about 30 years ago to eliminate the need for physical delivery of securities to settle trades. DTC retains physical custody of stock certificates on behalf of its members, which include all the major broker-dealers. Stock certificates for registered securities are deposited with the DTC and are held in the name of Cede & Co., DTC's nominee name. DTC records the

[50] Rule 203 of Regulation SHO imposes tighter borrowing and delivery requirements on short sellers, including new buy-in requirements for stocks with long delivery failures. Rule 203, which became effective January 3, 2005, should reduce strategic fails, but it may not eliminate them because it still provides for a market-maker exception and does not control short positions that occur 'ex-clearing', i.e., outside the National Securities Clearing Corporation.

[51] The clearing firm retains the cash proceeds from the short sale to secure the selling broker's delivery obligation. The clearing firm releases cash equal to the reduction in value of the shorted shares as the price of the shares declines (or demands additional cash margin if the share price rises).

[52] The rest clear and settle through mutual agreement of the brokers (and their clearing firm(s)).

transfer of securities by book entry; electronically it debits the seller's DTC account and credits the buyer's DTC account. No physical transfer ever occurs.

The NSCC was created in 1976 through the merger of three major clearing corporations (NYSE, AMEX, and NASD). NSCC works in conjunction with the DTC to provide centralized clearance and settlement for broker-to-broker stock trades in the United States. The NSCC clears and settles transactions through the Continuous Net Settlement (CNS) system. It guarantees completion of the transactions by assuming (a) the obligation of the buyers to pay for the shares upon delivery and (b) the obligation of the sellers to deliver the shares. During the trading day, the CNS continually nets all trades by its members in each security. The member's previous trading day's closing net long or short position is continually updated with the day's purchases and sales. At the end of the trading day, the member's updated net long or short position in each stock is communicated to the DTC for overnight processing.

Each short position is compared to the member's DTC account to determine if the member has enough shares on deposit to settle the short position. If so, then the DTC transfers the required number of shares from the member's DTC account to the NSCC's DTC account. Based on instructions from the NSCC, the DTC transfers shares received from members with short positions to the accounts of members with long positions. If the member with a short position does not have enough shares in its account to cover the short position, then the NSCC has five choices. It can wait another day to see whether the seller cures the fail by delivering the shares. Second, if it determines that the open short position is a high-priority obligation, it can attempt to arrange to borrow enough shares through its stock borrowing program to satisfy the open position (NSCC, 2003). If it is unable to borrow the shares, then the DTC has the three remaining choices: (a) it can demand a dealer buy-in (forcing the selling broker-dealer to buy the shares in the open market and deliver them to the DTC), (b) buy the shares itself in the open

market and charge the cost of the buy-in to the account of the seller, or (c) as a last resort, demand that the seller break the trade and compensate the buyer for the associated cost.

The NSCC's stock borrow program permits it to borrow shares from participating members to cover end-of-day open short positions that it deems to be of high priority. Addendum C-1 of the Rules and Procedures of the NSCC (2003) governs the operation of the stock borrow program. Members who wish to participate in the program inform the NSCC each day of the number of shares of each stock in their general unpledged account at the DTC which they are willing to lend. After the NSCC determines the number of shares it would like to borrow to satisfy all high-priority open positions, it applies a formula to determine from whom it will borrow the shares. The formula favors members who have the lowest stock loans from the NSCC and who pay the most clearing fees to the NSCC. When it borrows shares, the NSCC debits the lending member's DTC account but also credits that member with a long position in a special CNS sub-account set up specifically for the stock borrow program. The sub-account holds what is tantamount to an undated stock futures contract with the NSCC as the obligor. The NSCC also credits the lending member's regular CNS account with funds equal to the market value of the borrowed shares, which the lending member may invest overnight in an interest-bearing account. The DTC credits the borrowed shares to the NSCC's DTC account, which eliminates its short position, and transfers them to the buyer's DTC account. The buyer acquires all right, title, and interest in the borrowed shares – just as it would in any cash transaction that settles the regular way – including the right to vote the shares, receive dividends, resell them, or lend them (e.g., back to the NSCC through the stock borrow program). The NSCC charges a fee to each member with a short position that triggered the NSCC's need to use the stock borrow program. The NSCC returns the borrowed shares when it receives deliveries against outstanding short positions that exceed the amount of shares it needs to satisfy high-priority open short positions.

The stock borrow program can facilitate naked shorting in two ways. First, sellers can continue to fail to deliver because the NSCC can borrow the shares it needs to meet its clearing obligations through the stock borrow program. It does not have to force the seller who fails to deliver to buy in shares, nor does it have to go into the market to buy in the shares. It simply borrows them from another member firm to effect the buy-in. Since the NSCC covers the short position, the buyer of the stock also never has to buy them in.[53] Second, the stock borrow program allows the shares to be recycled. Each stock loan gives rise to another stock futures contract. Any single share could actually be relent multiple times, giving rise to multiple futures contracts. Each futures contract credited to a broker-dealer's sub-account at the DTC continues to be reported on the broker-dealer's books as a share held either in its proprietary account or in a customer account. In either case, the account holder believes he owns a real share with all the rights attached to it. Consequently, the stock borrow program effectively creates additional unauthorized shares of the issuer's stock. These undated stock futures contracts, which the financial press has referred to as phantom shares, inflate the amount of stock that is available for trading and also increase the amount of stock that is available for lending to short sellers (SEC, 2003b).[54]

4.2 Effect of Strategic Fails to Deliver

Naked shorting allows the manipulator to sell short profitably when borrowing shares would be expensive, for example, because of a small public float. It also enables the manipulator to conceal his true intent because he can mimic a market maker and need not borrow any shares.

[53] NSCC rules prohibit a member firm from buying in the shares in the open market. It must notify the NSCC if it wants a buy-in, and then the NSCC will attempt to effect the buy-in through the stock borrow program (NSCC, 2003).

[54] They also inflate the number of shares that are voted at the annual meeting. Apfel et al. (2001) cite the example of a proxy battle for control of Integrated Circuit Systems, Inc. in 1998. Twenty-two broker-dealers mailed proxies for more shares than they had in their accounts at the DTC, and the aggregate excess amounted to more than 10 percent.

The manipulator can achieve a pooling equilibrium in all four cases in Table 1. The manipulator sells shares short but she (or her clearing broker) does not arrange to borrow them. Her broker fails to deliver them to its clearing firm, who fails to deliver shares to the NSCC on the trade settlement date. Her side of the trade never settles with the NSCC, which records a fail to deliver in the clearing firm's account.[55] The NSCC can mitigate the effect of this delivery failure through its stock borrow program, which credits the buyer's clearing broker with the number of shares its customer purchased. The manipulator does not have to post collateral at the below-market rebate rate as he would if he had borrowed the shares, and he avoids the cost of searching for shares to borrow. However, the clearing house holds the cash proceeds from the sale of the shares. The manipulator's direct cost of shorting is zero. However, in order to effect the naked short sale, the manipulator needs the clearing broker's cooperation to maintain the naked short position for any significant length of time, for which the clearing broker will undoubtedly require some form of compensation. To simplify the problem, I ignore these indirect costs and assume C = 0 for the manipulator.[56]

Short Selling Is Expensive

First I consider the high-shorting-cost case (IV in Table 1). With a zero cost of shorting through strategic fails, the manipulator can profitably sell short at time 1 if $P^*(3)$ will be L even if the informed investor does not reduce his short sales.[57] But the manipulator must conceal his

[55] There are at least two ways the clearing broker can avoid this result. It can eliminate the naked short by lending the shares from the accounts of its margin customers or it can arrange with its clearing firm to borrow the shares, in both cases if sufficient shares are available. If they are, presumably the selling broker would have arranged the stock loan beforehand. I assume that these potential sources do not have enough shares.

[56] If these indirect costs are equal to C or greater, then my earlier results suggest that the manipulator will not be able to short additional shares profitably when the informed investor is shorting. Alternatively, naked shorting could be controlled by more effectively enforcing the short sale restrictions, as the SEC is seeking to do with Regulation SHO (SEC, 2003b, 2004).

[57] The manipulator can profitably sell short $(N[A-(1-\beta)L-\beta H]+(N+1)C)/(4B(N+1)) > 0$ shares. Since the manipulator can profitably enter the market even when the time 3 price will be L, the informed investor will realize a lower sale price for her shares due to the higher short sale volume. She will reduce her short selling (but not to zero) as long as she can still earn a profit. A sustainable equilibrium would require a simultaneous

short sales so that the active traders are not alerted. If $\beta \geq 1-p$, then the manipulator will not be able to cover his short sales at time 2 because the active traders will exit the market. Naked shorting abets this concealment enabling the manipulator to sell short

$$\hat{Q}_M^*(1) = \frac{N[A-(1-\beta)L-\beta H]+(N+1)C}{3B(N+1)} = \frac{N(H-L)[1-p-\beta]+(N+1)C}{3B(N+1)} \qquad (46)$$

shares at time 1. The informed investor will reduce her short sales to

$$\hat{Q}_I^*(1) = \frac{N[A-(1-\beta)L-\beta H]-2(N+1)C}{3B(N+1)} < \frac{N[A-(1-\beta)L-\beta H]-(N+1)C}{2B(N+1)} \qquad (47)$$

shares at time 1, which is fewer shares than the manipulator sells short. The manipulator can profitably sell $\hat{Q}_I^*(1)+\hat{Q}_M^*(1)$ shares if the time 3 price will be H because the informed investor does not sell short in that case, which achieves a pooling equilibrium. Naked shorting conceals the increase in the volume of shorting because fewer shares are borrowed. The active traders would still sell $\overline{Q}_A^*(2)$ shares in equation (35), and $P_M^*(2)$ in equation (36) would still be the manipulated price. $P^*(1)$ is lower as a result of the greater volume of short sales when there is naked shorting, and the lower cost of naked shorting and greater volume of shorting increase the manipulator's profit. Naked shorting drives the market price further below the stock's intrinsic value, and the difference is greater the lower is the perceived risk of manipulation.

$\hat{Q}_M^*(1)$ is directly related to N provided the manipulator can conceal the full extent of his naked shorting sufficiently so as not to drive the active traders from the market $(1-p>\beta)$.[58] An increase in the number of active traders reduces $P_M^*(2)$ and increases the profitability of manipulative naked short selling. But if the active traders' perception of the risk of manipulation

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solution of the informed investor's and the manipulator's short sale decision problems. The manipulator would sell more shares and the informed investor would sell fewer shares reflecting the manipulator's lower cost of shorting.

[58] This condition is equivalent to $1-p>q$, which is satisfied when there is no naked shorting, by the definition of β. It follows from equation (35) that $\overline{Q}_A^*(2)=0$ when $1-p=\beta$.

exceeds $1 - p$, for example, because they detect the naked short selling, then the active traders exit the market. Their departure removes the source of share sales that enables the informed investor and the manipulator to cover their short sales at time 2. As a result, the informed investor also exits the market. The manipulator could bid for shares from the uninformed investors, but that would presumably raise the share price above A and make manipulation unprofitable. The manipulator must disguise its trading, for example, by mimicking a market maker, to prevent the scheme from unraveling.

$\hat{Q}_M^*(1)$ is inversely related to β, the perceived risk of manipulation. $\hat{Q}_M^*(1)$ is directly related to H - L when $1 - p > \beta$. The manipulator's short sales increase (decrease) with the potential profit from manipulation (proxied by H – L) when the likelihood that the future stock price will be H is greater (less) than the likelihood that the manipulator will short the stock (the risk of manipulation). The stocks most likely to be affected by naked shorting are the riskier, small cap stocks that trade in the over-the-counter market. They have the greatest uncertainty about their true value (greater H – L), and the NASD's bid test restriction on short sales does not apply to Nasdaq SmallCap, OTCBB (over-the-counter bulletin board), and Pink Sheet stocks (SEC, 2003b, page 17), thus making it easier for a manipulator to enter the market (permitting greater q).

Is it possible for the manipulator to drive the informed investor out of the market? The greater volume of short sales could alert active traders to the greater risk that the market is manipulated. This greater perceived risk results in $\beta' > \beta$. The manipulator drives the informed investor from the market when the numerator in equation (46) is positive and the numerator in equation (47) is negative, that is, when

$$\frac{A - L - 2C}{H - L} - \frac{2C}{N(H - L)} \leq \beta' < \frac{A - L + C}{H - L} + \frac{C}{N(H - L)} \tag{48}$$

If β' is so much greater than β that the numerator in equation (46) becomes negative, neither the manipulator nor the informed investor will be able to sell shares short profitably. The active traders are so concerned about the risk of manipulation in that case that they will not accept a price $P^*(2)$ that would make short sales profitable at time 1. If they do not sell short at time 2 and the informed investor and the manipulator also do not, then the time 2 price is A and there will be no short sales at time 1.

On the other hand, keeping the active traders in the market maintains the scheme. Increasing N leads to lower $P^*(2)$, which leads to greater short selling by the informed investor and the manipulator and higher profits for both. The manipulator camouflages his short sales at time 1 to conceal their true purpose from the active traders. This provides a further incentive to naked short because trying to borrow the incremental shares could tip off active traders. By naked shorting, the manipulator appears to be a long seller. The manipulator may also spread negative news to reinforce the negative sell signal that the stock is overvalued in order to reduce β'.[59] Since the manipulator is short selling whether the future price will be H or L, he will spread negative news in both cases. As a result, the manipulator at times acts like an informed investor and at other times like a manipulator. It is difficult to discern his true motivation, for example, when investigating the possibility of market manipulation, without knowing the true intrinsic value of the shares. However, the manipulator will be more likely to engage in naked shorting when the future price will be L and the cost of shorting is high because incremental short sales of borrowed shares are unprofitable in that case. Naked shorting will be more prevalent as a fraction of total shorting the weaker the financial condition of the target firm.

Market makers can legally sell short more cheaply than other market participants, including naked shorting, provided it is in connection with bona-fide market-making (SEC,

[59] This activity is not manipulative if the future price will be L because the stock is overpriced at time 1 in that case.

2003b, 2004). They can conceal the true intent of their trading more easily than non-market-makers because of the market-maker exceptions to the short sale restrictions and because short selling by market makers is naturally greater for declining stocks. However, market makers usually like to end each day flat to avoid the risk of carrying overnight positions (Schwartz, 1991). Thus, unusually large or unusually lengthy fails to deliver in a stock are more consistent with manipulation than normal market-making.[60] Controlling naked shorting requires the regulators to enforce the short sale rules to ensure that manipulators do not register as market makers to exploit these exceptions.[61]

Short Selling Is Less Expensive

In the lower-shorting-cost cases (I – III), the manipulator can profitably short incremental shares at time 1 and time 2 when P*(3) will be L because the cost of naked shorting is zero. A pooling equilibrium can occur in which the behavior of the manipulator is indistinguishable from the informed investor's. The greater combined short sales by the informed investor and the manipulator at time 2 reduce $P^*(2)$, which causes the active traders to reduce their short sales. If the active traders perceive a greater likelihood of manipulation, they reduce their short sales. $P^*(2)$ is higher, and as a result, the informed investor and the manipulator increase their short sales at time 2.

4.3 Aggressive Naked Short Selling and Extended Fails to Deliver

Aggressive short selling is a bearish signal. Desai et al. (2002) find that a high level of short interest sends a strong negative signal because heavily shorted stocks experience significant negative abnormal returns during the period they are heavily shorted and have a higher

[60] In the Matter of Department of Enforcement v. John Fiero and Fiero Brothers, Inc., Decision, Before the National Adjudicatory Council, NASD, October 28, 2002.
[61] Rule SHO is intended to curb abusive shorting by requiring forced buy-ins of a stock when there are excessively large or excessively lengthy fails to deliver in a stock at the NSCC (SEC, 2004).

probability of delisting relative to the size and industry-matched control firms. Naked shorting intensifies these effects.

The manipulator will not sell short at time 2 in cases I-III if $P^*(3)$ will be H unless he can also manipulate the time 3 price. Recall that in the absence of manipulation, only a separating equilibrium is possible in case II. Naked shorting alters this situation. A manipulator can enter the market when the informed investor does not, mimic the informed investor at time 1 and time 2, and increase his profit by continuing to naked short the stock until its price is close to zero at time 3 when the price would otherwise be H. A similar result applies in case III. With such aggressive naked shorting, the manipulator no longer needs to rely on active traders because he does not have to cover his short position. In fact, he can earn greater profit by driving them from the market to eliminate their competition for short sales at time 2.

Pooling Equilibrium

The informed investor will not sell short when the intrinsic value is H. The manipulator can achieve a pooling equilibrium in case II by short selling $Q_M(1) = Q_I(1)$ shares at time 1 and $Q_M(2) = Q_I(2)$ shares at time 2 when the intrinsic value is H to mimic the informed investor. The manipulator can conceal his manipulation in a pooling equilibrium.

The active traders sell short $\overline{Q}_A^*(2)$ shares at time 2 in equation (44) and the time 2 price will be $P_M^*(2)$ in equation (45). The manipulator can sell additional shares short at time 3 to drive down the price. Due to the concealment that naked shorting affords, the other market participants might misinterpret a sharp drop in share price, even to pennies a share, as the result of informed investors selling the shares of a firm whose profitability and business prospects have deteriorated, rather than manipulation. When information is revealed that suggests that the intrinsic value is H, the uninformed traders' demand curve at that time is $H - BQ$ since A = H

when p = 0.[62] If the manipulator covers his short position, the price rises to H, and he suffers a loss. To avoid the unraveling problem, the manipulator can depress $P^*(3)$ by extending existing fails to deliver and naked shorting an additional $H/B - (Q_I(1) + Q_I(2))$ shares at time 3.[63] The manipulator's profit is

$$\pi_M = P^*(1)Q_M(1) + P^*(2)Q_M(2) - P^*(3)[Q_M(1) + Q_M(2)]$$

$$= \frac{A^2 - (L + 2C)^2}{4B} + \frac{(A - L + 2N\beta(H - L) - 2NC)^2}{4B(N + 2)^2} + (L + C)\left(\frac{A - L + 2N\beta(H - L) - 2NC}{2B(N + 2)}\right) \quad (49)$$

when $P^*(3) = 0$. The manipulator's incentive is to drive the firm's share price as close to zero as possible. If he only does so when the future price will be H, his trading never interferes with the informed investor's. He can maintain this short position beyond time 3 either by extending the fail to deliver or by rolling over the naked short position (by repurchasing the naked shorted shares and simultaneously naked shorting the same number of shares to a different dealer). In that case, the Equity Trade Journal for the stock will report daily trading volume but the NSCC Continuous Net Settlement Report will reveal large open short positions and significant persistent net fails to deliver but the DTC Weekly Position Report may indicate little, if any, daily changes in the net positions of the shorting dealer(s) at the DTC.[64]

[62] The price is H if all the short positions are covered. If they are not, the market price will have to drop below H to induce the uninformed traders to purchase additional shares.

[63] The SEC documented such manipulative behavior in the case of SEC v. Rhino Advisors, Inc. and Thomas Badian (SEC, 2003a).

[64] The daily trading volume could be quite high if the manipulator is rapidly turning over its short position. The National Association of Securities Dealers compiles the Equity Trade Journal, which reports the details of daily trading in each stock on a trade-by-trade basis. The Continuous Net Settlement Report compiled by the NSCC provides a daily summary of the trade settlement activity in each stock at the clearing house on a dealer-by-dealer basis. The Weekly Position Report compiled by the DTC based on the Continuous Net Settlement Report shows the changes in position dealer-by-dealer and day-by-day. But for the open short position, the daily trading and settlement activity may appear to be normal market making because the dealer's net position on the day does not change. Pumping the trading volume also reduces the short interest ratio (short interest divided by the average daily trading volume) to help conceal the manipulation.

Building a short position of H/B to drive $P^*(3)$ to zero would involve naked shorting more shares than the firm has outstanding because H/B > (A – L)/B.[65] The manipulator can not drive the share price close to zero unless he can naked short an extraordinary number of shares.[66] This form of manipulation would result in a precipitous drop in the firm's share price to well below its intrinsic value, unusually heavy trading volume, and unusually large and persistent fails to deliver at the NSCC. Preventing this activity requires the clearing house to enforce its buy-in rules for fails to deliver and to impose penalties on short positions that are rolled over for an extended period, which is the purpose behind new Regulation SHO (SEC, 2004).

Alternate Pooling Equilibrium

A different pooling equilibrium is also possible with more aggressive naked shorting that takes place when the intrinsic value of the stock at time 3 would be L. In this case, the manipulator will compete with the informed investor by selling short $Q_M(1)$ shares at time 1 and $Q_M(2)$ shares at time 2. The informed investor reduces his short sales, but the overall volume of short sales is greater because of their combined shorting. The manipulator sells short $Q_I(1) + Q_M(1)$ shares at time 1 and $Q_I(2) + Q_M(2)$ shares at time 2 if the time 3 intrinsic value is H. He can drive $P^*(3)$ close to zero by increasing his naked short position to L/B.[67] The manipulator's profit depends on his ability to manipulate the firm's stock price and to keep it depressed. The stronger the financial condition of the firm at time 3 (the higher is L), the greater the number of shares the manipulator has to sell short at time 3 to drive the price close to zero.

[65] For example, on January 15, 2005, Charter Communications, a Nasdaq National Market stock, had 36,600,000 outstanding shares other than shares held by insiders and a reported short interest of 88,520,000 shares. Source: Bloomberg, LP. Details are available on request from the author.

[66] The NASD reported that Charter Communications had short interest of 88,520,000 shares in January 2005, but Charter reported having outstanding shares minus shares held by insiders of only 36,600,000 shares.

[67] The uninformed investors' demand curve at time 3 is $L - BQ$ if $P^*(3) = L$ since A = L when p = 1. Details are available on request from the author. Even if the manipulator's short position is L/B, it might exceed the entire number of shares the firm has outstanding.

The manipulator earns greater total profit in this alternate pooling equilibrium because he can profitably manipulate the market even when the future price will be L and also because he can substitute his own short sales when the informed investor cuts back. The firm's share price is close to zero at time 3 regardless of its intrinsic value. The volume of manipulative short selling is greater due to the manipulator's heavier short sales. The manipulator's potential profit is greater, but so is the risk of detection because of the greater naked short sales volume, the larger fails to deliver at the clearing house, and the adverse reaction of the informed investor when the manipulator interferes with his trading.

If the manipulator has driven the price close to zero to avoid the unraveling problem, will the informed investor or active traders reenter the market and buy the underpriced shares? They will not unless they are confident they can overcome the impact of the manipulation. However, if they offer to buy shares, the manipulator can increase the naked short position to prevent the share price from rising. If he can keep the price close to zero, then prospective financiers are likely to conclude that the firm's prospects have worsened and refuse to lend or invest in its equity. Customers may cease doing business with it as well because its warranties will appear worthless. Eventually, the firm will exhaust its liquidity and have to file for bankruptcy. The manipulator will be relieved of its obligation to cover its short position if the firm's shares are cancelled in bankruptcy.[68] This scenario leads to a zero cost of covering the short positions. This form of manipulation may involve a single manipulator or a group of manipulators who act in concert and make an unusually high percentage of apparently unlucky equity investments that become worthless in bankruptcy, all of which have unusually high trading volume, large and persistent fails to deliver, and a precipitous drop in share price below the stock's intrinsic value (often to just pennies a share).

[68] House Report (1991). In most reorganizations (and in all liquidations), the plan of reorganization (liquidation) calls for the cancellation of the debtor's common shares.

Separating Equilibrium

Finally, the manipulator might be even more aggressive if he is not satisfied with the profit he can earn in a pooling equilibrium. But the more aggressive naked shorting increases the risk of detection. If the active traders detect the manipulation, two reactions are possible. They might exit the market (N = 0), which leads to case II regardless of the informed investor's cost of short selling. The manipulator benefits because there is no competition from active traders to sell short at time 2, which results in higher P*(2). He sells short to uninformed traders at time 1 and time 2.

Alternatively, the informed investor might free ride on the manipulation. He benefits if the manipulator drives $P^*(3)$ below L. However, he risks being branded a manipulator. The informed investor will not free ride if the intrinsic value is H because he would then become a manipulator.

Suppose the informed investor and the active traders decide to free ride on the manipulation. Their short sales reduce P*(2) and thus the manipulator's profit. For large N, the informed investor sells short approximately (A – 2C)/(3B) and the manipulator sells short approximately (A + C)/(3B) shares at time 1, neither sells short any shares at time 2, and the active traders collectively sell short approximately (A + 4C)/(3B) shares at time 2. $P^*(2)$ is approximately zero. Competition among the active traders drives $P^*(2)$ toward zero and results in none of them earning a nontrivial profit. The manipulator's profit if the informed investor and a large number of active traders free ride is

$$\pi_M = (A+C)^2/(9B) \tag{50}$$

The informed investor earns a smaller profit because he sells short fewer shares. The combined profit of the manipulator and the informed investor is

$$\pi_M + \pi_I = (2A-C)(A+C)/(9B) \tag{51}$$

The manipulator maximizes his profit by driving competing short sellers from the market. The manipulator aggressively sells short enough shares at time 1 to drive the price down sharply. The heavy volume of sell orders and resulting sharp decline in price are likely to scare off legitimate traders if they signal that the shares are about to be delisted to the OTC Bulletin Board market, which is much less liquid than the Nasdaq National Market and the exchanges. This likelihood increases if the manipulator is recognized by the informed investor and the active traders as an aggressive 'bear raider'. In this case, the manipulator not only does not want to conceal his identity from legitimate active traders, he wants to make them aware of it to drive them out of the market. He will try to maintain his naked short position until the shares are ultimately cancelled in liquidation. He maximizes his short sale proceeds by naked shorting A/3B shares at time 1, A/3B more at time 2, and either L/B – 2A/(3B) or H/B – 2A/(3B) more at time 3. The market prices are P*(1) = 2A/3, P*(2) = A/3, and P*(3) is close to zero, all regardless of whether $P^*(3)$ should be H or L. The manipulator's profit is

$$\pi_M = A^2/(3B) \qquad (52)$$

which is greater than π_M in equations (50) and (51) for reasonable values of C.

4.4 Complementary Manipulative Trading Strategies

The manipulator can depress the price to reduce the market value of the firm's float and discourage the informed investor and active traders from trading the stock. It also increases the probability of delisting (Desai et al., 2002). The firm will fail to meet both the NYSE's and the Nasdaq National Market's listing standards if its share price falls below one dollar for 30 consecutive trading days.[69] Pushing the stock to the Nasdaq SmallCap market or the OTC Bulletin Board market will discourage institutional investors from trading the stock because of the lower liquidity in those markets. But it will remove the constraint on short selling that the

[69] The American Stock Exchange does not have a minimum share price standard but does have a minimum required value of the outstanding shares, which will be violated if the firm's share price drops low enough.

tick test and the bid test impose and allow the short seller greater latitude in depressing the share price further. Selecting the stocks of riskier, small cap firms to manipulate makes it easier to cause this failure because the drop in share price is less likely to arouse suspicion and attract arbitrageurs who might bid up the price and cause the scheme to unravel. Pushing the stock onto the Nasdaq SmallCap or the OTC Bulletin Board (OTCBB) also reduces the risk of regulatory detection if the securities regulators focus their attention on the exchanges and the Nasdaq National Market.[70]

Which alternative the manipulator pursues depends on whether he wants to conceal his behavior and his aversion to detection risk. He can reduce this risk by achieving a pooling equilibrium but must sacrifice some of the potential profit from more aggressive manipulation. Departures from a pooling equilibrium (e.g., more aggressive naked shorting) are more likely when the manipulator registers as a market-maker to avail himself of the short sale rule exceptions and also when he succeeds in driving the share price below one dollar to push the stock into a less regulatory intensive market, such as the OTC Bulletin Board. Driving the price down enough to trigger delisting provides a natural cover for the manipulation because it signals a deterioration in the firm's financial condition and its prospects to those market participants who are unaware of the fraud.[71]

The type of behavior modeled in this section is more likely to occur in the OTC Bulletin Board market than on the exchanges or in the Nasdaq National Market. It might take the form of trading by investment 'pools' (informal investor networks) that have enlisted the cooperation of one or more market-makers, who reduce the risk of detection because of the market-maker exceptions to the short sale rules. These pools could reduce the risk of detection by spreading the

[70] The weaker short sale restrictions on the OTCBB, including under Regulation SHO, indicates the difference in regulatory environment.
[71] This pattern of behavior is documented in the SEC's enforcement action against manipulative short sellers in SEC v. Rhino and Badian (SEC, 2003a).

short selling across a large number of sophisticated investors, and the large number of sellers would increase the credibility of the sell signal. These pools could include investors, such as hedge funds who profess to follow short-sale strategies, who regularly sell stocks short because interspersing manipulative trades with regular short selling reduces the risk of detection.[72]

4.5 Regulation SHO

Regulation SHO imposes a mandatory buy-in for stocks that remain on the NSCC's threshold list for 13 consecutive trading days (SEC, 2004). There is no market-maker exception. If all stock trades cleared through the NSCC, the buy-in requirement would prohibit fails to deliver in excess of 0.5 percent of a stock's outstanding shares for more than 18 trading days (5 days to make the list plus 13 on it). Since trades also clear outside the NSCC (referred to as *ex-clearing* in the securities industry), greater and more extended fails are still possible. Research concerning whether Regulation SHO has curbed excessive fails to deliver seems warranted. Is there evidence of buy-ins of stocks that have remained on the threshold list for 13 days, or has Regulation SHO raised the proportion of stock trades that settle ex-clearing?

5. Floating-Price Convertibles and Resolving the Unraveling Problem

Floating-price convertibles (FPCs) became a popular form of PIPE (private investment in public equity) financing in the 1990s. Hillion and Vermaelen (2004) identify 467 issues of FPCs that firms issued between January 1995 and July 1998. FPCs allow the manipulator to resolve the unraveling problem by covering his short position with FPC conversion shares. This can increase the profitability of manipulative short selling. Because he faces less risk of a short squeeze, the manipulator can sell short more shares. The greater selling further reduces the firm's share price, which increases the manipulator's profit. It also increases the number of conversion shares at no

[72] Such pools, if they exist, would be reminiscent of the 'bear pools' of the 1930s and earlier eras (Stedman, 1905, Bernheim and Schneider, 1935, Wyckoff, 1968, and House Report, 1991).

additional cost. Finally, the manipulator obtains a valuable delivery option. If he naked shorts, he does not have to drive the firm into bankruptcy to avoid covering the short position; he can deliver the conversion shares.

5.1 Equilibrium When Short Selling Is Expensive

The firm needs to raise F to finance a new project. It sells F = P*(1) x Z face amount of FPCs at time 1, which is convertible into Z new shares at a discount to the market price at the time of conversion. The discount D is usually in the range from 15% to 25% (Hillion and Vermaelen, 2004). The initial conversion ratio is Z/(1 – D), and the intrinsic value of the conversion option is initially F/(1 – D). Conversion can begin after a specified grace period expires, which I assume is time 2.

I assume that the firm invests the proceeds of the new issue in a zero-net-present-value project. The present value of the project's free cash flow is F/(1 – D) to compensate for the discount (and leave the wealth of the existing shareholders unaffected).[73]

The manipulator buys the FPCs and sells short at time 1. His short selling at time 1 reduces the firm's share price and increases the initial conversion ratio for given new issue proceeds F. Since the announcement of a zero-NPV project does not shift the active traders' demand curve, the demand curves are $P^*(1) = A - B(\hat{Q}_I(1) + \hat{Q}_M(1) + Q_M(1))$ at time 1 and $P^*(2) = A - B(Q_M(1) + \overline{Q}_A^*(2))$ at time 2. The manipulator converts a fraction E of the FPCs at time 2 and the rest at time 3.

[73] Investing in a zero-NPV project does not affect its share price or the demand curve for its shares. If the present value of the free cash flow were only F, the financing would dilute the firm's equity. Its share price is A before it announces the financing but $A' = A - DFB/(A - L)(1 - D)$ following the announcement. In that case, the firm's share price will decrease when it announces an FPC issue, which is consistent with the market reaction to the announcement documented by Hillion and Vermaelen (2004).

The manipulator sells short $\hat{Q}_M(1)$ shares at time 1. In a pooling equilibrium, the manipulator sells short at time 1 and covers the short position at time 2 if the time 3 intrinsic value is H. His profit on his FPC investment apart from any short sales is

$$\pi_M = DF/(1-D) \tag{53}$$

which is the value of the discount specified in the FPC contract. It does not matter whether the FPC investor converts at time 2 or time 3 so long as the conversion shares are priced and delivered concurrently because the FPC pricing formula provides a perfect hedge. The FPC investor receives F/((1 – D)P) shares, which are worth P each and F/(1 – D) in the aggregate. Including short sales, his profit is

$$\pi_M = DF/(1-D) + \hat{Q}_M(1)[P^*(1) - P^*(2) - C] \tag{54}$$

The manipulator earns the same profit (equation (40)) on his short sales as he would without the FPC investment. The short sales do not affect the profitability of the FPC investment because the FPC provides a perfect hedge when the conversion shares are priced and delivered concurrently. The manipulator should not use the conversion shares to cover the short position because that strategy would save him P*(2) at time 2 but cost him H > P*(2).

Recall that when the time 3 intrinsic value is L, the manipulator can not profit at the margin unless he can achieve a lower cost of shorting than the informed investor or unless he can pursue strategies that cause the informed investor to reduce his short sales. Incremental short selling is unprofitable when the manipulator's cost of shorting is C. The manipulator's opportunity cost of using the conversion shares to cover the short position is P*(2), and a profit-maximizing informed investor will short sell that number of shares which reduces the marginal

shorting profit to zero when the cost of shorting and covering is P*(2) + C. The manipulator can not increase his overall profit by substituting conversion shares for market share repurchases.[74]

Investing in the FPC gives the manipulator a valuable delivery option because he can use conversion shares to cover the naked short position if the clearing house demands delivery of shares and if it is more expensive to purchase shares in the market than to deliver conversion shares. Merely submitting the conversion demand notice eliminates the naked short position.[75] This option reduces the manipulator's risk of detection because a short squeeze is impossible so long as the number of conversion shares exceeds the naked short position. Investing in the FPC also generates a virtually guaranteed profit because of the discount and the FPC's built-in hedge. This profit is available to offset potential losses on the short selling, if the price of the shares should rise. The FPC provides insurance that reduces the manipulator's financial risk.

5.2 Equilibrium When Short Selling Is Inexpensive

The manipulator can profitably sell short incremental shares at time 1 and time 2 in the lower-shorting-cost cases (I – III) when the time 3 intrinsic value is L. A pooling equilibrium results in which the manipulator behaves like the informed investor. If he buys FPCs, he obtains the valuable delivery option and insurance against loss. The greater combined short sales by the informed investor and the manipulator at time 2 reduce $P^*(2)$, which causes the active traders to reduce their short sales. The greater the perceived risk of manipulation (β), the greater the informed investor's and the manipulator's short sales at time 2 and the smaller are the active traders' short sales.

5.3 Aggressive Naked Shorting

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[74] The manipulator can avoid paying $\hat{Q}_M(1)P^*(2)$ to repurchase shares in the market at time 2 if he uses $\hat{Q}_M(1)$ conversion shares to cover the short position. That strategy reduces the face amount of the FPCs by $\hat{Q}_M(1)P^*(2)[1-D]$ and the profitability of the FPC investment by $\hat{Q}_M(1)P^*(2)$.

[75] Current securities regulations treat a short sale as a covered transaction when the short seller enters into a commitment to buy or secure delivery of enough shares to meet its delivery obligation.

The manipulator will not sell short in cases I-III if the intrinsic value is H unless he can manipulate $P^*(3)$. The manipulator might engage in aggressive naked shorting and use the FPC conversion shares for either of two purposes, depending on the anticipated time 3 price of the stock. I assume that through his due diligence, the manipulator can accurately assess the firm's future prospects well enough to determine whether the manipulation-free time 3 price will be L or H. This assumption is more plausible when the manipulator purchases FPCs because of the due diligence that usually accompanies a private securities issue (Blackwell and Kidwell, 1988).

FPCs give the manipulator two valuable overlapping options, the option to expropriate wealth from the other shareholders and the option to gain voting control of the firm. He can aggressively short the stock to depress its price if the time 3 intrinsic value is L and then cover the short position with the cheap conversion shares. He uses naked shorting to exploit an overvalued stock, whose price would normally be expected to fall to L. He can permanently manipulate the share price below L, perhaps to only pennies a share, and keep the price artificially depressed by flooding the market with the cheap conversion shares to dilute the share price and by extending his strategic fails.

Separating Equilibrium When the Shares Are Overvalued

The issuance of FPCs is perceived as a negative signal if active traders and the informed investor associate it with manipulation. They can either stay out of the market to avoid becoming victims of the manipulation resulting in a separating equilibrium or else free ride on the manipulator's behavior resulting in a pooling equilibrium.

If the manipulator is the only short seller, he enhances his profit by naked shorting. The manipulator's profit is

$$\pi_M = -F + P^*(1)\hat{Q}_M(1) + P^*(2)\hat{Q}_M(2) + (F/[(1-D)P^*(t)] - \hat{Q}_M(1) - \hat{Q}_M(2))P^*(3) \qquad (55)$$

when the manipulator engages in naked shorting at time 1 and time 2, tenders the conversion notice at time 3, and fully covers both short positions at time 3. P(t) is the conversion price, which is determined at the date t the FPC investor submits the conversion notice. The manipulator could increase his profit by $(\hat{Q}_M(1) + \hat{Q}_M(2))P^*(3)$ by extending the strategic fail until the firm liquidates. Since the firm is overvalued at time 1, its share price would decline from P*(1) to L in the absence of any manipulation. The manipulator maximizes his profit by maximizing his short sale proceeds and naked shorting $P^*(3)$ close to zero.[76] The manipulator should sell short A/(3B) shares at time 1 resulting in P*(1) = 2A/3 and Z=3F/(2A). These short sales occur at the time of the financing because the manipulator has an incentive to begin shorting the stock immediately. He should sell short an additional A/(3B) shares at time 2. If L > A/3, the manipulator will naked short more shares than the firm has outstanding. The share price is P*(2) = A/3, which is half what it was at time 1. At time 3, the financial condition of the firm is revealed, and the uninformed traders' demand curve shifts to P = L – BQ. The manipulator drives $P^*(3)$ close to zero by adjusting the short position to L/B shares. If L > 2A/3, the manipulator will increase his short position; otherwise, he will decrease it.[77]

The manipulator's profit is

$$\pi_M = DF/(1-D) + A^2/(3B) \qquad (56)$$

the profit on its FPC investment plus its profit on the short sale. Aggressive short selling has increased the manipulator's profit. This has occurred with a huge volume of naked shorting – possibly exceeding the firm's outstanding shares – and a precipitous decrease in share price that first cut the price in half and then reduced it close to zero.

[76] The manipulated price could be less than a penny if market convention allows quotations in a fraction of a cent, as for example the OTCBB market does.

[77] For example, if L = A/3, then the manipulator would naked short the same number of shares the firm has outstanding, doubling the float. He would halve the short position at time 3 to A/(3B), restoring it to what it was at time 1. The strategic fail substantially decreases at the same time the share price declines because the intrinsic value of the shares decreases.

Suppose instead the manipulator submits the conversion notice at time 2. By selling short an additional 2A/(3B) shares for a total of A/B, he can depress $P^*(2)$ close to zero and obtain a virtually infinite number of shares. He dilutes the equity ownership interest of the other shareholders virtually to zero, covers the naked short at time 3 at virtually zero cost, and realizes profit equal to

$$\pi_M = DF/(1-D) + 2A^2/(9B) + L(A-L)/B \qquad (57)$$

which equals the profit on the FPC plus the profit on the naked short sale at time 2 plus the entire equity value of the firm at time 3. The profit in equation (57) is greater if $L(A-L)/B > A^2/(9B)$, that is, the value of the firm's equity at time 3 exceeds the profit on the time 2 short sale when the manipulator waits to time 3 to submit the conversion notice. This condition is satisfied if the likelihood that the time 3 intrinsic value of the firm will be H is high enough.

The manipulator selects the conversion strategy that maximizes his profit. If the time 3 intrinsic value is (highly likely to be) L, the manipulator will sell short A/(3B) shares at time 2, cover the short at time 3, and wait until time 3 to depress the share price close to zero. If the time 3 intrinsic value is (highly likely to be) H, he will sell short A/(2B) shares at time 2 to depress the price close to zero at time 2.[78]

For example, suppose the manipulator realizes at time 2 that the firm's share price the next period will be H, rather than L as originally expected, say, due to favorable developments in the firm's business. Suppose further that the securities regulators or the clearing house require all securities dealers to clear up all fails to deliver.[79] The manipulator would face potentially large

[78] A = (1 – p)H + pL. The manipulator selects the strategy based on the relative values of L(A – L)/B and $A^2/(9B)$, which depends on the value of p. A smaller value of p, and hence greater likelihood of H, favors L(A – L)/B and the strategy of driving the share price toward zero at time 2.

[79] They might impose large financial penalties or prohibit further short selling if any fails to deliver exceed an acceptable duration. In the case of the Sedona stock manipulation, the NASD ordered the NSCC to require all

losses on his short sales. By short selling an additional 2A/(3B) shares at time 2, he can drive the share price close to zero. He would obtain sufficient shares to dilute the equity ownership interest of the other shareholders virtually to zero. He realizes virtually the entire equity value of the firm and achieves a profit equal to

$$\pi_M = DF/(1-D) + 2A^2/(9B) + H(A-L)/B \tag{58}$$

Owning the FPCs gives the manipulator a valuable hedge against the regulatory risk that the regulators might enforce, or require the clearing house to enforce, the restrictions on short sales and fails to deliver. The FPCs give the manipulator a valuable manipulative tool he can use to blunt the impact of any unexpected tightening of short sale restrictions.

Pooling Equilibrium When the Shares Are Overvalued

The informed investor may decide to free ride on the manipulation when he realizes that the stock is being manipulated by an aggressive short seller. He recognizes that sellers will be able to use the decrease in the stock's price ex post to justify their short selling. The justification will appear plausible if as a result of the manipulation, the company is driven out of business. If the informed investor and the manipulator sell short at time 1, the active traders will also at time 2, which will intensify the downward price pressure on the stock.

The manipulator maximizes his profit by naked shorting, which distinguishes his behavior from the informed investor's. The manipulator sells short

$$Q_M^*(1) = [(3N+2)A + (2N+1)C]/[(9N+7)B] \tag{59}$$

and

$$Q_M^*(2) = (N+1)[A + (3N+4)C]/[(9N+7)B] \tag{60}$$

The informed investor sells short fewer shares unless its cost of shorting is zero, which occurs

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dealers to settle all fails to deliver within 10 days (SEC, 2003a). Regulation SHO provides such a restriction. See section 2.

when it joins in the naked shorting. The total short sales at time 1 are approximately 2A/(3B) – 5C/(9B) when the number of participating active traders is large and the market price is

$$P^*(1) = (N+1)[3A+5C]/[9N+7] \tag{61}$$

which approaches A/3 + 5C/9, about one-third the initial price, as a limiting value. The total short position at time 2 amounts to approximately A/B, which is more shares than the firm has outstanding. The manipulator and the informed investor each accounts for about 4/9 of the total (3/9 sold at time 1 and 1/9 at time 2), and the active traders collectively account for 1/9 (all shorted at time 2). The time 2 market price is

$$P^*(2) = [A+(3N+4)C]/[9N+7] \tag{62}$$

which approaches C/3 as a limiting value as the number of active traders increases.

Separating Equilibrium in Which the Manipulator Gains Control

FPCs embody a valuable option to gain voting control, which the manipulator has an incentive to exercise when the time 3 price will be H. The manipulator can gain control by acquiring one more than 50 percent of the outstanding shares. The informed investor will not sell any shares short because the time 3 price will be H. The manipulator naked shorts at time 1 and time 2 to depress the share price before submitting conversion notices. He then demands conversion shares based on the cheap price, and if the price is low enough, he obtains sufficient shares to achieve voting control of the firm with the economic benefits that accompany control.

The manipulator's profit-maximizing strategy is to submit the conversion notice at time 2 after driving down the price. The conversion price is P*(2)[1 – D]. His profit is

$$\pi_M = \frac{DF}{1-D} + \frac{[H-P^*(2)]F}{P^*(2)[1-D]} + Q_M(1)[P^*(1)-2C-H] + Q_M(2)[P^*(2)-C-H] \tag{63}$$

Profit is equal to the value of the discount plus the profit on the cheap conversion minus the losses on the short sales at time 1 and time 2 (since H > P). The profit on the cheap conversion is

greater the lower is $P^*(2)$, but the loss on the short sales at time 2 is also greater. Naked shorting reduces these losses. The manipulator will not sell short at time 1 to avoid the losses on time 1 short sales.[80] In that case, the active traders would not sell short because they do not observe any short sales at time 1.

Suppose the manipulator did not sell any shares short at time 2. His profit would be

$$\pi_M(Q_M(2)=0) = F[H - A(1-D)]/[A(1-D)] \tag{64}$$

The manipulator makes money on the FPC without short selling but short selling increases his profit. The manipulator increases his profit by increasing his short sales to A/B.[81] P*(2) falls nearly to zero, and the manipulator gets enough shares to dilute the other shareholders' collective equity interest virtually to zero. The manipulator realizes virtually the entire economic value of the firm even after using A/B shares to cover his short position. The firm's share price will remain near zero because of the dilutive effect of the conversions at a near-zero price. The manipulator's profit is approximately

$$\pi_M = [H(A-L)/B + F] - F - AC/B = H(A-L)/B - AC/B \tag{65}$$

The manipulator is able to expropriate wealth amounting to H(A – L)/B from the other shareholders.

5.4 The Firm's Failure to Honor Conversion Notices to Control Moral Hazard Risk

How can the firm deal with the moral hazard risk that the FPC investor turns out to be a manipulator who aggressively short sells its stock? The firm can attempt to control this risk by insisting on a provision in its FPC contract that prohibits the investor from short selling its stock. The investor should not object because the floating-price feature of the FPC provides a natural

[80] He also has an incentive to manipulate the share price upward before the FPC pricing. If he can manipulate the price above H + 2C, he is assured a profit on short sales at this price, possibly large enough to cover the cost of the FPC purchase.

[81] π_M in equation (63) is convex to the left of $Q_M(2) = A/B$.

hedge against a falling share price. The firm can also refuse to honor the investor's conversion requests. However, that strategy is risky for the firm because it exposes the firm to the accusation that it has breached the FPC contract.[82]

If the firm fails to honor any of the conversion requests, the investor is out of pocket F but still has an investment that is in theory worth F/(1 – D). FPCs are usually unregistered, and so there is no public market for them. The FPC investor can attempt to find another private investor who is willing to purchase the FPCs. But if the FPC investor has manipulated the firm's share price downward and the firm has refused to honor the conversion requests, a prospective sophisticated private purchaser would be alerted by the decline in share price. He would presumably discover in the course of his due diligence, for example by calling the firm, that it has refused to honor the conversion notices and would refuse to purchase the FPCs. The FPC investor would be unable to exit his investment. This prospect should discourage manipulators from purchasing FPCs with a short-sale prohibition in the contract. By insisting on such a provision, a prospective FPC issuer can detect a potential manipulator.

5.5 FPCs and the Faulty Contract Design Hypothesis

Hillion and Vermaelen (2004) find that the stocks of FPC issuers realize an average return of -43.78 percent after adjusting for market effects during the year following the offering and that 85 percent of the post-announcement returns are negative. They test three hypotheses that might explain this behavior and find support for the faulty contract design and last-resort financing hypotheses but no support for the undervaluation hypothesis.

The sharp price declines Hillion and Vermaelen document are consistent with the model of share price manipulation developed in this paper. My model suggests the following

[82] There have been dozens of lawsuits filed by firms alleging that the FPC investors manipulated their shares through naked short selling and other manipulative devices. In many of these cases, the FPC investors countersued accusing the firm of breach of contract for failing to honor their conversion notices.

explanation for their results. Manipulators offer potentially attractive long-term financing to smaller, riskier firms who may be unable to raise such large amounts of equity by issuing common stock or conventional fixed-conversion-price convertible securities and issue FPCs as a last resort. The financiers, who can drive the harder bargain because of the firm's need for funds and lack of alternatives, insist that the firm issue FPCs. Optimistic managers and existing shareholders, who may believe that their firm's shares are more likely to rise than fall when the funds are invested in the business (thereby reducing the number of shares the firm will have to issue when the FPCs are converted), fail to appreciate the risks inherent in issuing FPCs and agree to the choice of security, possibly even issuing it multiple times (Hillion and Vermaelen, 2004). Once the firm has issued the FPCs, the financiers/manipulators can exploit its flaws to manipulate the stock to their advantage and to the firm's detriment.

In light of the SEC's concerns about the impact of naked shorting and the implications of my model that FPCs are an effective instrument to complement naked shorting, it would seem appropriate to reexamine the after-issue performance of FPC issuers to determine whether (and to what extent) naked shorting (and other forms of manipulation) might have contributed to the large negative returns. Such an investigation might reveal that the faulty contract design hypothesis is an even better explanation for these returns than Hillion and Vermaelen suggest because the flaws in the contract facilitate manipulative short selling.[83]

[83] I have investigated the post-issue performance of all the public firms I could find that issued FPCs and that were also associated with XBank, a financier widely known to be associated with FPC financing as an investor, as a financial advisor, or in some other investment banking capacity. Based on an EDGAR search, I identified 102 public firms that met these criteria. The FPCs were issued between January 2, 2000 and May 31, 2004. The share prices of all 102 firms have fallen relative to a control group since the firms issued FPCs, 47 of the firms have either gone out of existence or their shares are no longer quoted (even in the Pink Sheets), 16 of the shares are quoted at less than a penny, 26 others are quoted at less than a dollar, and the other 13 are still trading at prices above a dollar. Sixty-five of the stocks were delisted within a year of the FPC issue. Three of the firms are now under the control of the FPC financiers. The research is not complete but it at least suggests that XBank has a remarkable record when it comes to working with FPC issuers. Details are available on request from the author.

6. Conclusion

The SEC's recent adoption of Regulation SHO has drawn attention to the potentially disruptive impact of manipulative short selling, and in particular, naked short sales masquerading as routine fails to deliver. An interesting empirical question concerns the impact of manipulative short selling on the capital market in the United States. Some recent evidence suggests that strategic fails to deliver are pervasive and are not confined to the over-the-counter market. The impact of these fails to deliver has never been investigated, however. A related question concerns the impact of Regulation SHO, which became effective January 3, 2005, and in particular, whether this regulation has had the intended effect of curbing abusive short sales.

Analyzing both questions requires a model of stock market behavior. This paper models market equilibrium and describes the market impact of manipulative short selling. Naked short selling is shown to be a particularly effective and damaging manipulative device. It is difficult to control because of the market-maker exceptions to the normal restrictions on short selling, including permitting naked short selling in the course of bona-fide market-making activity. A recent securities innovation called floating-price convertible securities removes an important constraint on short selling by resolving the unraveling problem. I conclude that the current capital market environment and regulatory regime, Regulation SHO notwithstanding, are conducive to manipulative short selling. I will leave it to future research to test this conclusion empirically and measure the impact of the manipulation.

A recent study of FPCs by Hillion and Vermaelen finds that FPC issuers experience large negative returns following the issue, which the authors partly attribute to flaws in the FPC contract. The SEC's concerns about the impact of naked shorting, as reflected in its adoption of Regulation SHO, coupled with the implications of my model that FPCs can intensify the adverse impact of naked shorting suggest that further study of Hillion and Vermaelen's faulty contract

design hypothesis is warranted. This research should investigate whether FPC investors short the issuer's stock, for example, in anticipation of submitting conversion notices. Such a strategy is unnecessary because of the natural price hedge built into the floating-price structure. Evidence of significant shorting would imply that the FPC's faulty contract design may play a more important role in the share price decreases Hillion and Vermaelen document.

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Table 1

Description of the Market Equilibrium

This table describes the market equilibrium for alternative values of the cost C of selling shares short.

Case	Cost of Selling Short (C)	Short Sales by the Informed Investor at Time 1: Repurchase at Time 3 $(Q_I(1))$	Short Sales by the Informed Investor at Time 1: Repurchase at Time 2 $(\hat{Q}_I(1))$	Stock Price at Time 1 $(P^*(1))$	Short Sales by the Informed Investor at Time 2 $(Q_I(2))$	Short Sales by Active Traders $(\bar{Q}^*_A(2))$	Stock Price at Time 2 $(P^*(2))$
I	$C=0$	$\frac{A-L}{2B}$	0	$L+\frac{A-L}{2}$	$\frac{A-L}{2B(N+2)}$	$\frac{N}{N+2}\frac{A-L}{2B}$	$L+\frac{A-L}{2(N+2)}$
II	$0<C\le\frac{A-L}{2N}$	$\frac{A-L-2C}{2B}$	0	$L+\frac{A-L}{2}+C$	$\frac{A-L-2NC}{2B(N+2)}$	$\frac{N}{N+2}\frac{A-L+4C}{2B}$	$L+\frac{A-L+4C}{2(N+2)}$
III	$\frac{A-L}{2N}<C\le\frac{(3N+3)(A-L)}{2N^2+5N+5}$	$\frac{(A-L)(3N+2)-(2N^2+5N+3)C}{B(4N+3)}$	$\frac{(2NC-A+L)(N+1)}{B(4N+3)}$	$L+\frac{(A-L)(2N+2)+(3N+3)C}{4N+3}$	0	$\frac{N}{4N+3}\frac{A-L+(2N+3)C}{B}$	$L+\frac{A-L+(2N+3)C}{4N+3}$
IV	$C>\frac{(3N+3)(A-L)}{2N^2+5N+5}$	0	$\frac{N(A-L)-(N+1)C}{2B(N+1)}$	$L+\frac{(N+2)}{2N+2}(A-L)+\frac{C}{2}$	0	$\frac{N}{N+1}\frac{A-L}{B}$	$L+\frac{A-L}{N+1}$

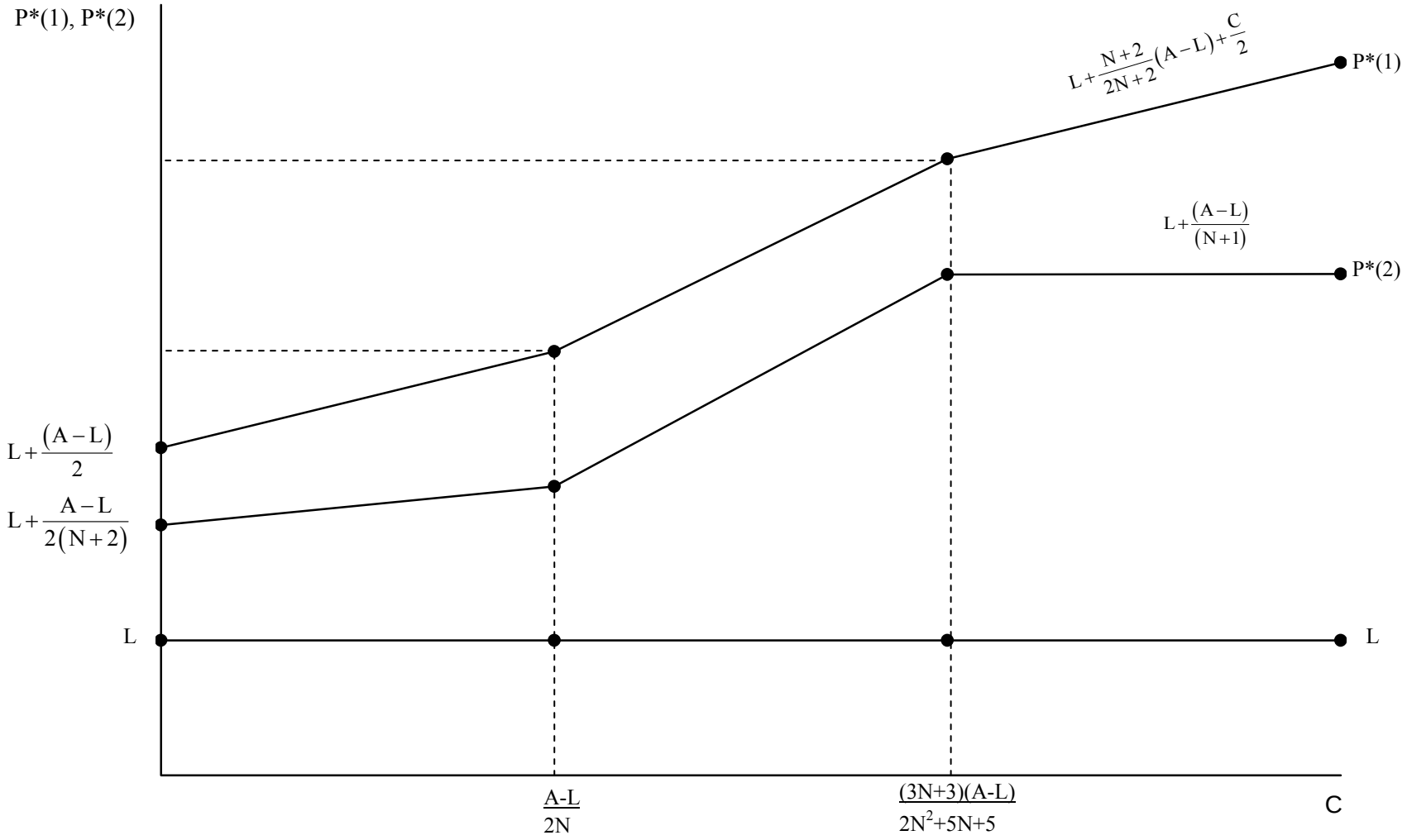

Figure 1 The Equilibrium Market Prices

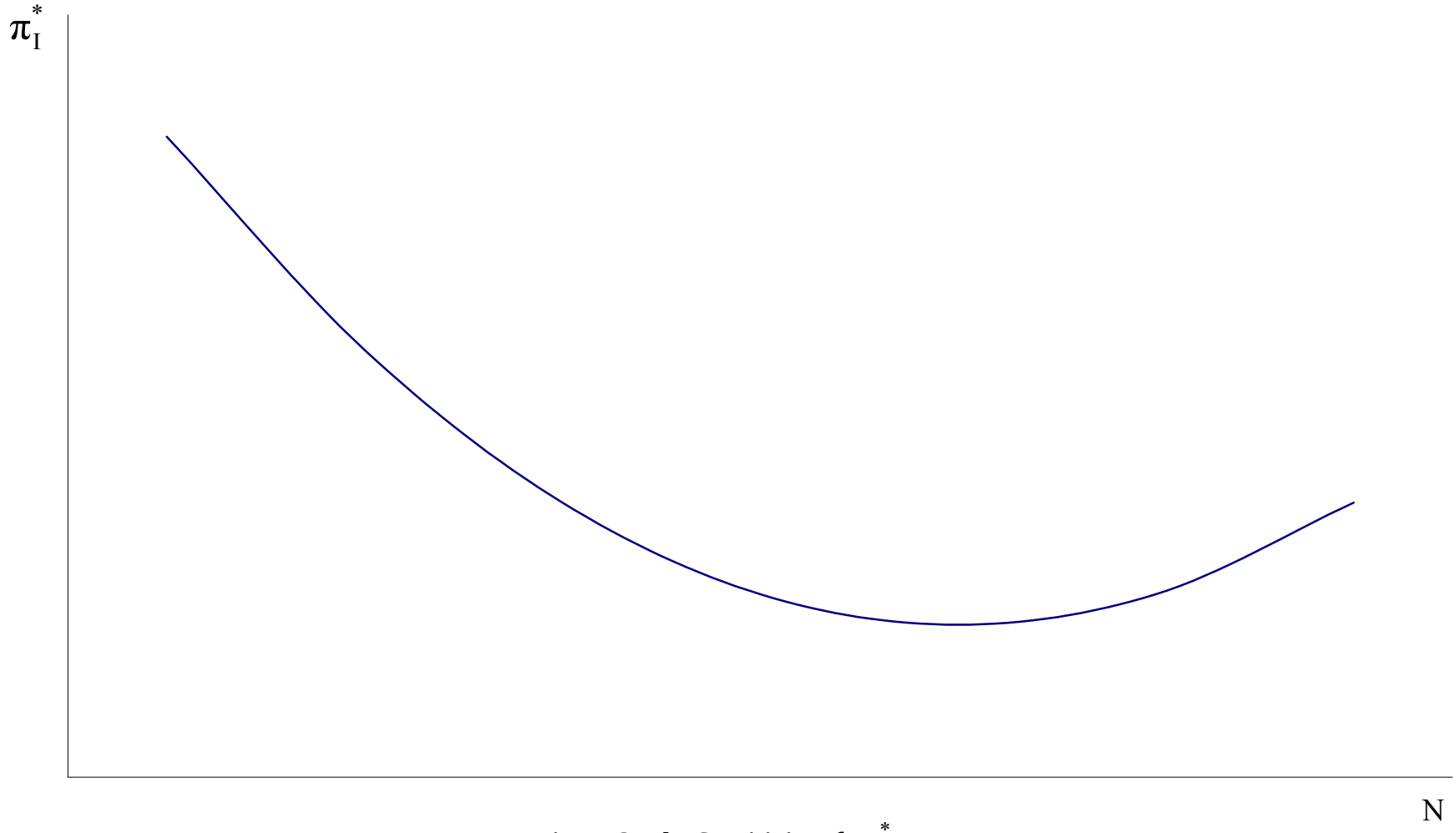

Figure 2 The Sensitivity of π_I^* to N